Simulation Technologies for Hybrid Dynamic Systems

Pieter J. Mosterman
Real-time and Simulation Technologies
The MathWorks, Inc.
Natick, MA
pieter_j_mosterman@mathworks.com
http://www.xs4all.nl/~mosterma

Hybrid Dynamic Systems

- Continuous Behavior In Modes, α
  - dynamic behavior

$$f_{\alpha_i}(\dot{x}, x, u, t) = 0$$

  - algebraic constraints

$$g_{\alpha_i}(x, u, t) = 0$$

- Transition From Mode α_i to α_{i+1}

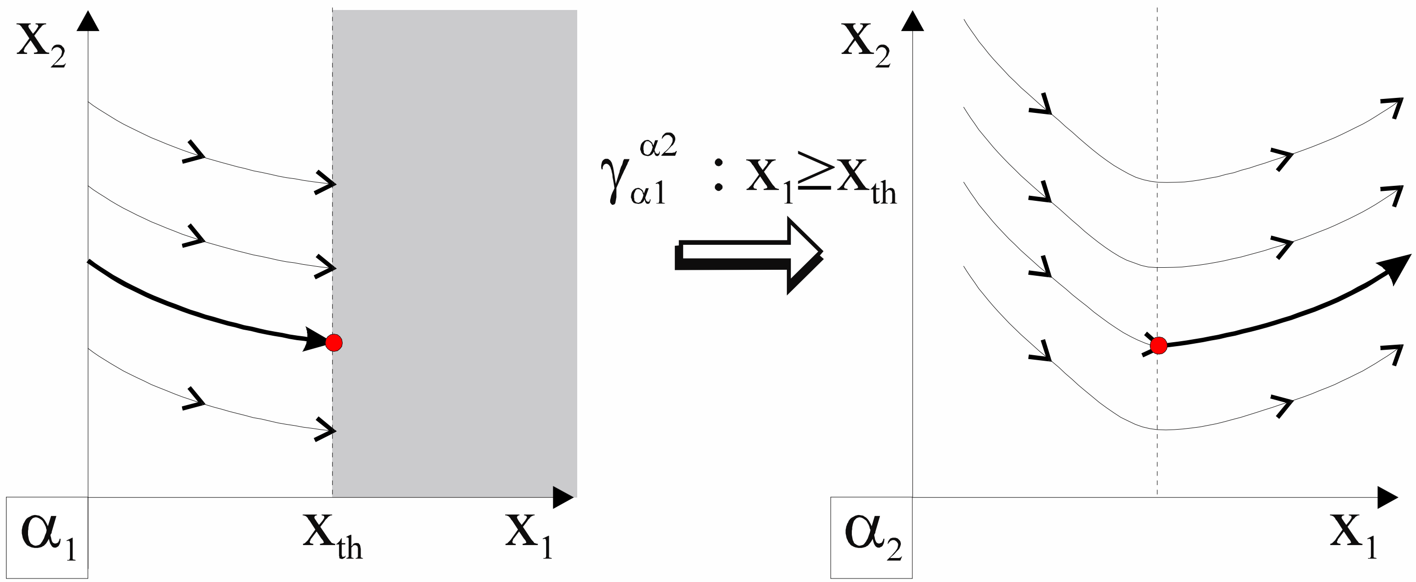
$$\gamma_{\alpha_i}^{\alpha_{i+1}}(x, u, t) \geq 0$$

- Explicit reinitialization

$$f_{\alpha_i}(\dot{x}, x, u, x^-, t) = 0$$

Phase Space Behavior

- State Vector, x, Spans Phase Space
- Each Mode, α, Has Phase Space Behavior
- Transition Regions Are Defined by, γ

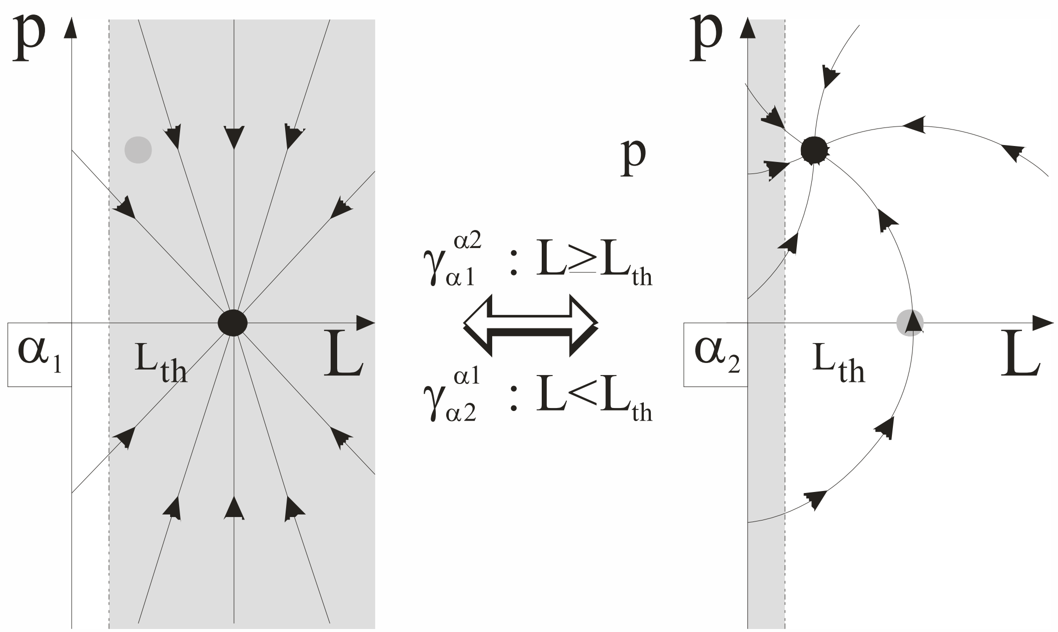


Continuous Behavior Simulation

- Sorting
  - identify and sort the set of equations
- Solving
  - manipulate the system of equations to achieve an acceptable index
- Initial state calculation
  - calculate the initial value of the variables
- Numerical integration
  - solve the differential and algebraic equations (DAEs) in time

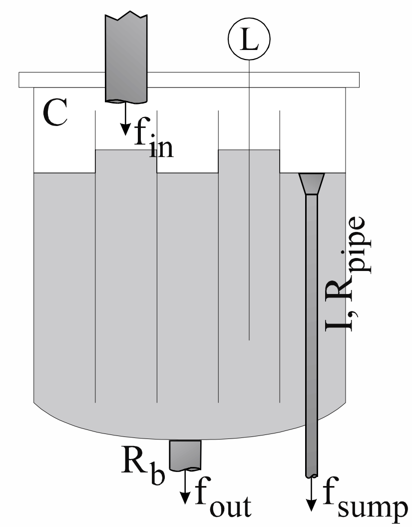
Phase Space Behavior Example

- An Evaporator Vessel
- State Vector, x, Consists of Level, L, and Momentum, p
- Mode Dependent Steady State



$\gamma_{\alpha 1}^{\alpha 2} : L \geq L_{th}$

$\gamma_{\alpha 2}^{\alpha 1} : L < L_{th}$

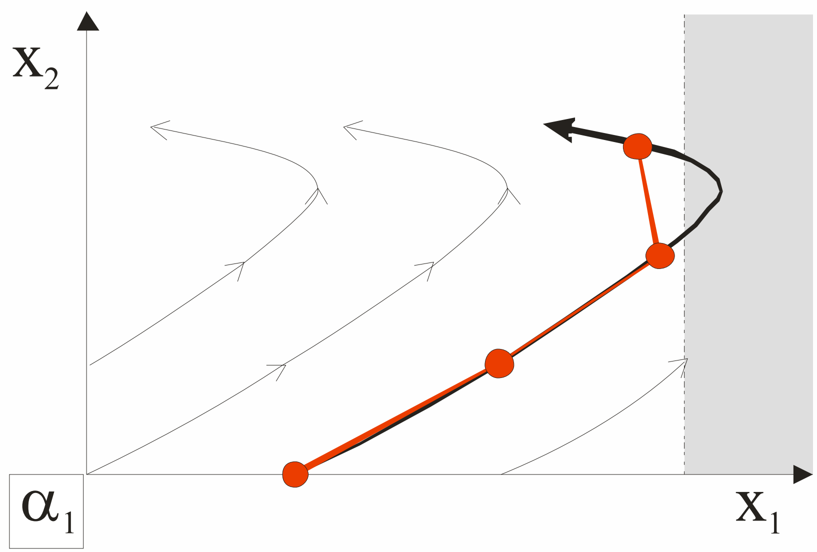


Event Handling

- Sign Change of the Mode Transition Function, γ, Has to Be
  - detected
  - located

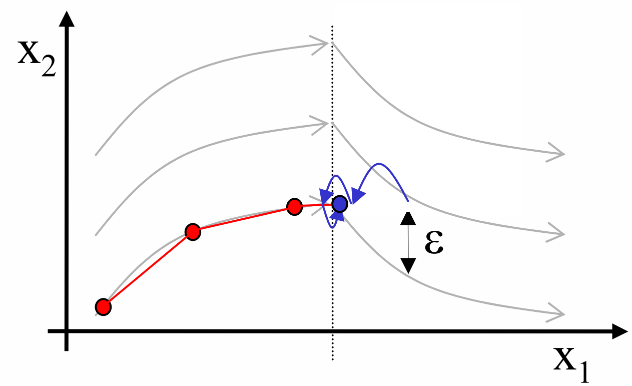
1. State Event Detection

- Detection
  - requires odd number of zeros in integration interval
  - add mode transition equations to dynamic behavior equations



2. State Event Location

- Continuous Variables Cross Thresholds
  - mode switch
  - locate switching point
  - typically by iteration, e.g., bisection



- Standard Approach
  - fix mode changes of discontinuities
  - make a regular integration step determined by system dynamics
  - detect and locate crossings in this interval
    - polynomial interpolation for efficiency

Prevent Stepping Across Boundary

- Assume step index k is real (Esposito, Kumar & Pappas, 2001)

$$\frac{d\gamma}{dk} = \left(\frac{\partial\gamma}{\partial x}\frac{dx}{dt}\right)\frac{dt}{dk} \Rightarrow \frac{d\gamma}{dk} = \left(\frac{\partial\gamma}{\partial x}f\right)h(k)$$

- Select

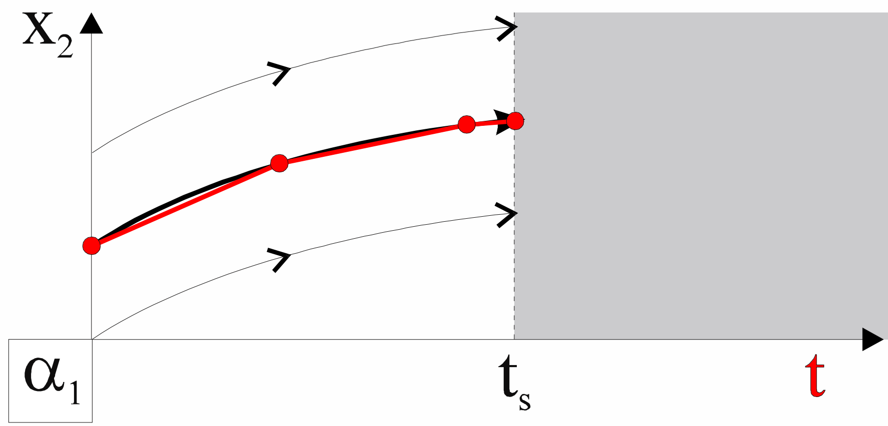
$$h(k) = -\eta\frac{\gamma(k)}{\frac{\partial\gamma}{\partial x}f}$$

- Exponential Convergence $\gamma(k) \longrightarrow 0$

$$\frac{d\gamma(k)}{dk} = -\eta\gamma(k)$$

3. Time Event Handling

- Crossing Is Known *A Priori*, Allows Efficient Implementation
  - no iteration required



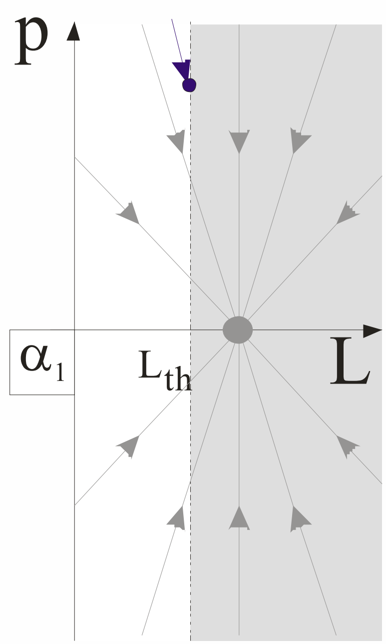
- Typical Examples
  - discrete event systems
  - control signals

Solver Effects

- After a Mode Switch, Solver Must Be Reinitialized
  - history of integration points
  - step size, tolerances, …
- Interaction Between Discrete and Continuous Part
  - many discrete state changes may not affect the continuous model part
    - integrated circuit modeling (VHDL-AMS)
  - repeatedly resetting the solver degrades performance, though
  - simulate discrete and continuous parts separately with roll-back mechanism to handle occasional interaction

4. Chattering

- A Number of Mode Transitions May Follow One Another Rapidly
- For Example, Overflow Level Selection Results in:



p

$\gamma_{\alpha 1}^{\alpha 2} : L \geq L_{th}$

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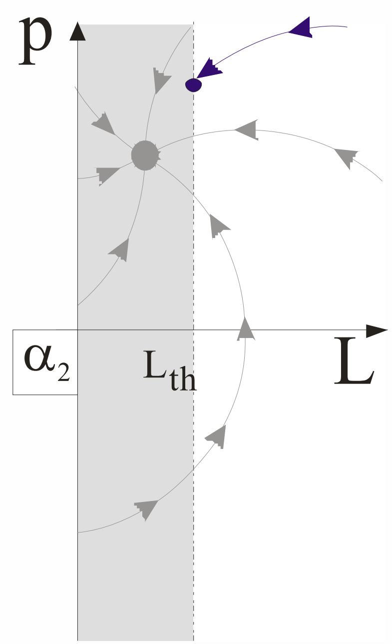
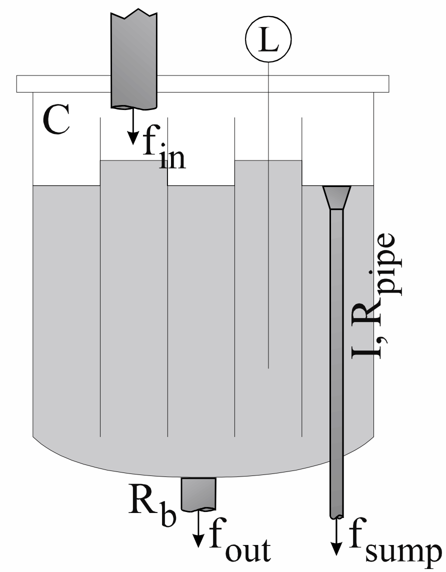
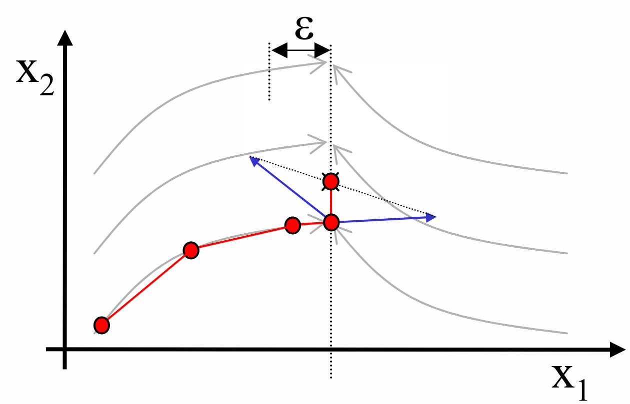




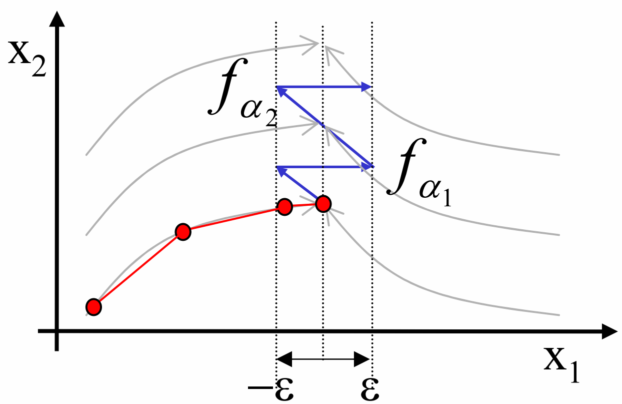
Preserve Slow Dynamic Behavior

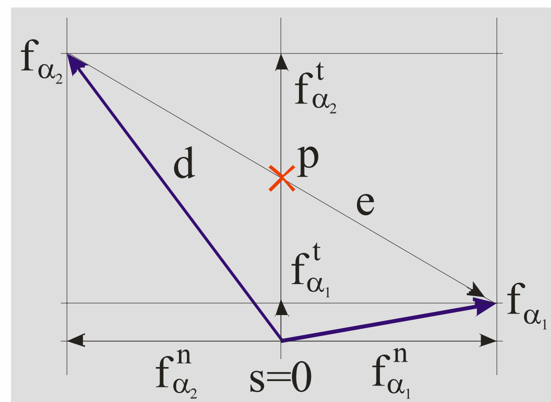
- Immediate Switch Back
  - chattering between two modes
  - remove fast chattering behavior
- Equivalent Dynamics Have to Be Found
  - in two-dimensional space by Filippov construction using instantaneous field vectors at the switching surface



Physical Rationale

- Models With Chattering Behavior Arise Because Higher Order Physical Phenomena Have Been Neglected
  - e.g., fluid inertia, hysteresis in overflow behavior in evaporator

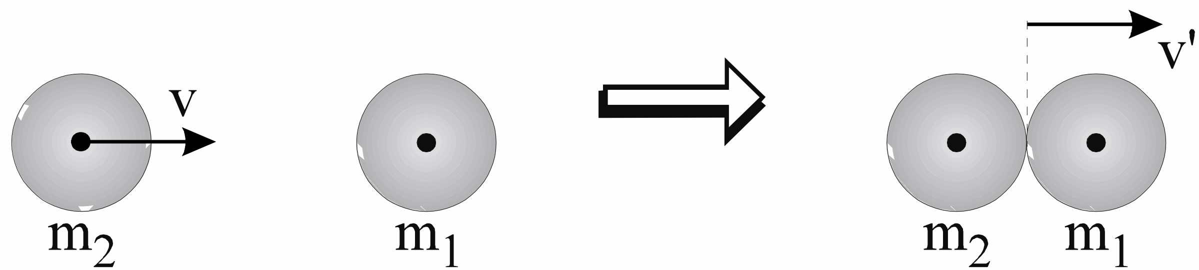




$$v = \frac{\delta x}{\delta t_{\alpha_1} + \delta t_{\alpha_2}} = \frac{\left(f^t_{\alpha_1}\frac{\varepsilon}{f^n_{\alpha_1}} + f^t_{\alpha_2}\frac{\varepsilon}{f^n_{\alpha_2}}\right)}{\left(\frac{\varepsilon}{f^n_{\alpha_1}} + \frac{\varepsilon}{f^n_{\alpha_2}}\right)}$$

Index Changes

- When The System Of Equations Changes, Algebraic Manipulations May Be Required To Reduce Complexity (*Index*)
- Perfect Nonelastic Collision



- Two Issues
  - get into subspace
  - remain in subspace

5. Run-Time Equation Solving

- Differential Equations

$$f : \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

- Algebraic Equations
  - Moving Freely, α_1

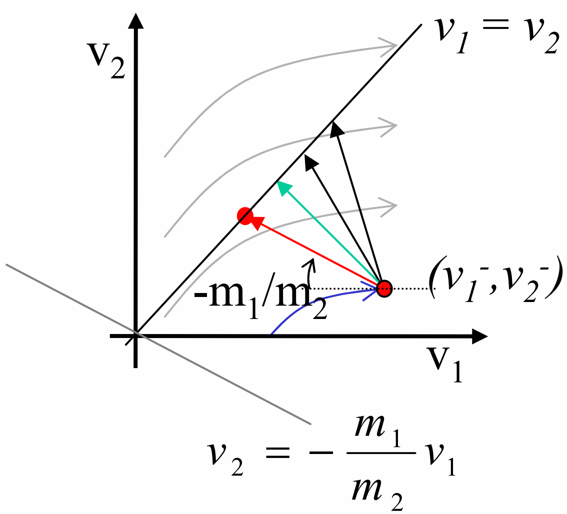
$$g_{\alpha_1} : \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

  - Contact, α_2,Reduces Order

$$g_{\alpha_2} : \begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} -F_2 \\ v_2 \end{bmatrix}$$

State Space Changes

- Inelastic Collision Between Two Masses
- Contact Behavior Is on a Manifold, g_{α_2}



$$f : \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$g_{\alpha_1} : \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$g_{\alpha_2} : \begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} -F_2 \\ v_2 \end{bmatrix}$$

6. Implicit Jumps from Algebraic Constraints

- Projection Can Be Derived By Integration

$$F_1 = -F_2 \Rightarrow$$

$$m_1 \dot{v}_1 = -m_2 \dot{v}_2 \Rightarrow$$

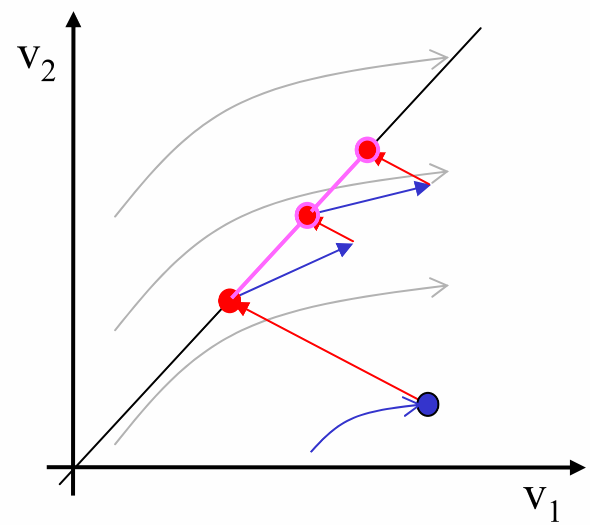
$$m_1 v_1 + m_2 v_2 = m_1 v_1^- + m_2 v_2^- \Rightarrow$$

$$v_2 = -\frac{m_1}{m_2} v_1 + \frac{m_1 v_1^- + m_2 v_2^-}{m_2}$$

- Physical Principles
  - *parameter abstraction*
  - conservation of momentum

7. Dynamic Behavior in Subspace

- Field Is Directed Away From Subspace
- Repeated Projections



- Implemented in
  - Simulink to study induction motor design
  - HYBRSIM for hybrid bond graph simulation

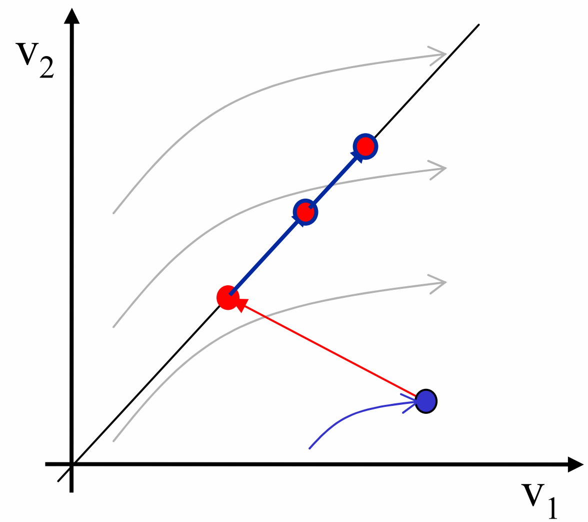
Use Reduced Dynamic Behavior in Subspace

- Compute Dynamics Using Differentiation (Pantelides, 1988)
- For Colliding Bodies Differentiate $v_1 = v_2$

$$F_1 = F_{1,i} + F_{1,x}; F_2 = F_{2,i}; F_{1,i} = -F_{2,i} \Rightarrow$$

$$\left.\begin{aligned} m_1\dot{v}_1 + m_2\dot{v}_2 &= F_{1,x} \\ \dot{v}_1 &= \dot{v}_2 \end{aligned}\right\} \Rightarrow$$

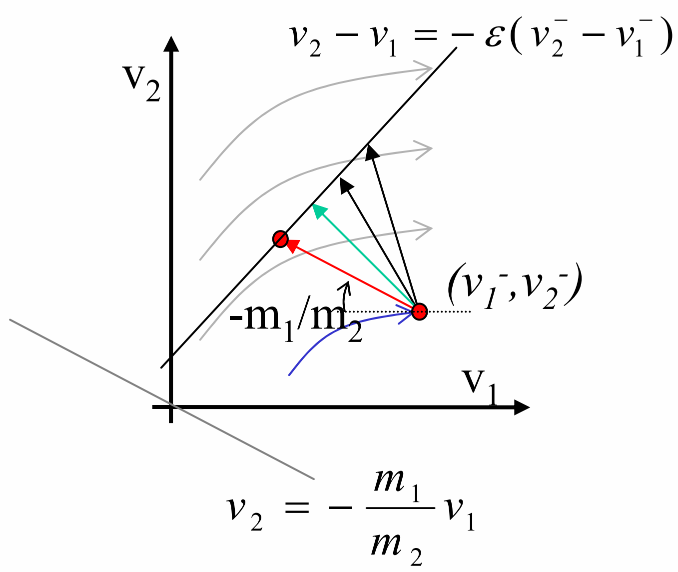
$$\dot{v}_1 = \frac{F_{1,x}}{m_1 + m_2}$$



- Numerical Drift
  - projections nevertheless (physically inconsistent, though)

8. Explicit Reinitialization

- An Elastic Collision Between Two Masses



$$f: \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$g_{\alpha_1}: \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$g_{\alpha_2}: \begin{bmatrix} F_1 \\ v_2 - v_1 \end{bmatrix} = \begin{bmatrix} -F_2 \\ -\varepsilon(v_2^- - v_1^-) \end{bmatrix}$$

Explicit Jumps from A Priori State

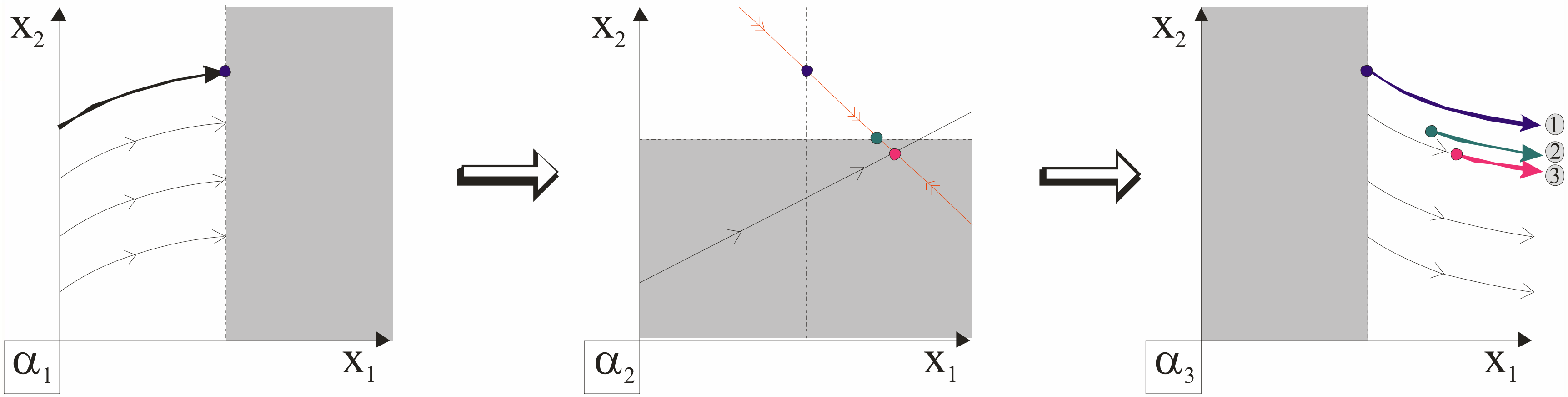
- Elastic Collision
  - *time scale abstraction*
  - constraints active at a point in time

$$g_{\alpha_2} : \begin{bmatrix} F_1 \\ v_2 - v_1 \end{bmatrix} = \begin{bmatrix} -F_2 \\ -\varepsilon(v_2^- - v_1^-) \end{bmatrix}$$

- System Behavior Is on a Manifold, $g_\alpha(x, x^-)$
  - allow use of a priori state, x^-
  - critical to define a priori state
  - immediate further mode change is required!

9. Sequence of Mode Changes

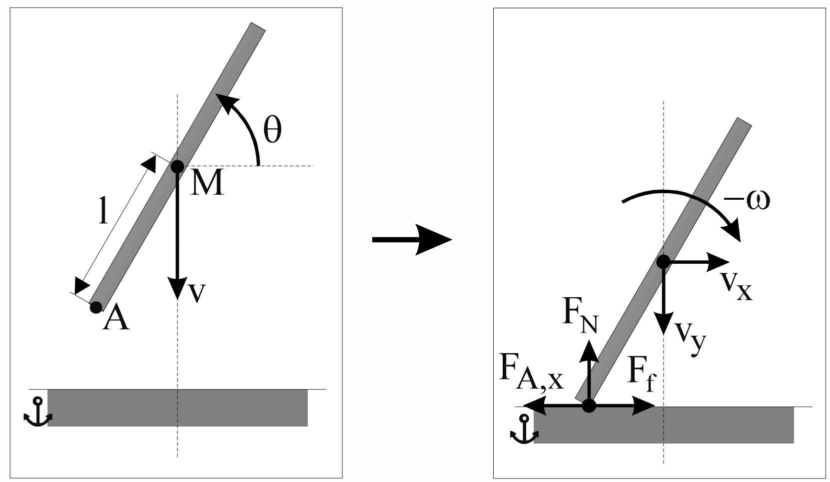
- One Mode Change May Be Immediately Followed By a Consecutive Mode Change



- What Is the Correct State Vector in the Final Mode?
  - ‘time scale abstraction implies completed projection’
  - ‘parameter abstraction implies *invariance of state*’

Colliding Rod

- Thin Rigid Rod Collides With Rigid Surface With Coulomb Friction (Lötstedt, 1981)
  - free, α_0
  - stuck, α_1
  - sliding, α_2



- Mode Transitions
  - collision:

$$\gamma_{\alpha_0}^{\alpha_1} : y_A \leq 0$$

  - sliding:

$$\gamma_{\alpha_1}^{\alpha_2} : |F_{A,x}| > \mu F_N$$

State Invariant Iteration

- If the New Velocities Cause Sliding Then Use State Variable Values in Previous Real Mode
- Otherwise, in Degenerate Case $\mu = 0$, Stuck Mode Causes

$$v_{A,x} = 0$$

- Because $v_x = -\omega \; l \sin\theta$, This Causes

$$v_x \neq 0$$

When the Rod Starts to Slide

- But, There Is Never a Horizontal Force!

10. Impulse Handling

- Non-Smooth Behavior May Cause Impulses
  - numerical approximation may cause inaccuracies
- For Example, Colliding Rod
  - transition to sliding $|F_x| > \mu F_N \Rightarrow |F_x| > \mu (F_y - F_g)$
  - contains impulses, P, $|P_x| > \mu (P_y - F_g) \Rightarrow |P_x| > \mu P_y$
  - using impulse areas $|v_x - v_x^-| > \mu (v_y - v_y^-)$
  - instead of numerical approximation

$$\frac{|v_x - v_x^-|}{\varepsilon} > \mu \left(\frac{v_y - v_y^-}{\varepsilon} - F_g\right)$$

Overview of Hybrid Simulation Phenomena

- State Event Handling
  - detection
  - location
- Time Event Handling
- Chattering
- Run-Time Equation Manipulation
- Jumps in Impulse Space
  - implicit
  - explicit as a function of the a priori state
- Sequence of Mode Changes
  - state update
  - aborted projection
  - state invariant
- Impulse Handling

Software Packages Evaluated

- χ, University of Eindhoven
- ABACUSS, Massachusetts Institute of Technology
- BaSiP, University of Dortmund
- DOORS, University of Magdeburg
- Dymola, Dynasim
- gPROMS, Imperial College, London
- HYBRSIM, DLR Oberpfaffenhofen
- Omola, Lund Institute of Technology
- SHIFT, University of California, Berkeley
- Simulink, The MathWorks, Inc.
- Smile, GMD FIRST Berlin
- 20-sim, University of Twente

1999	Event detection	Event location	Time event	Chattering	Equation manipulation	Implicit jumps	A priori state	Invariant state	Aborted projection	Impulse handling
χ	√	√	√		√		√			
ABACUSS	√	√	√		√		√			
BaSiP	√	√								
DOORS	√	√	√		√		√			
Dymola	√	√	√		√		√			
gPROMS	√	√	√		√		√			
HyBrSim	√	√			√	√	√	√		√
Omola	√	√	√		√		√			
SHIFT	√						√			
Simulink	√	√	√				√			
Smile	√	√	√		√		√			
20-sim	√	√	√				√			

Conclusions

- Hybrid Simulation Introduces a Number of Phenomena
- (Most of) These Phenomena and Possible Solutions Have Been Described in Literature
- State of the Art Simulation Packages Only Address a Subset

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  - Hubertus Tummescheit, and
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Clarification of Pathological Cases

- Chattering
  - infinitely small steps in time
  - moves past any given point in time (locally)
- Divergence of Time
  - discrete steps of zero time
  - time remains the same (does not diverge)
- Zenoness
  - steps of non-zero time
  - does not move past a limit point in time
    - converging series