

Discrete-event modelling and diagnosis of quantised systems

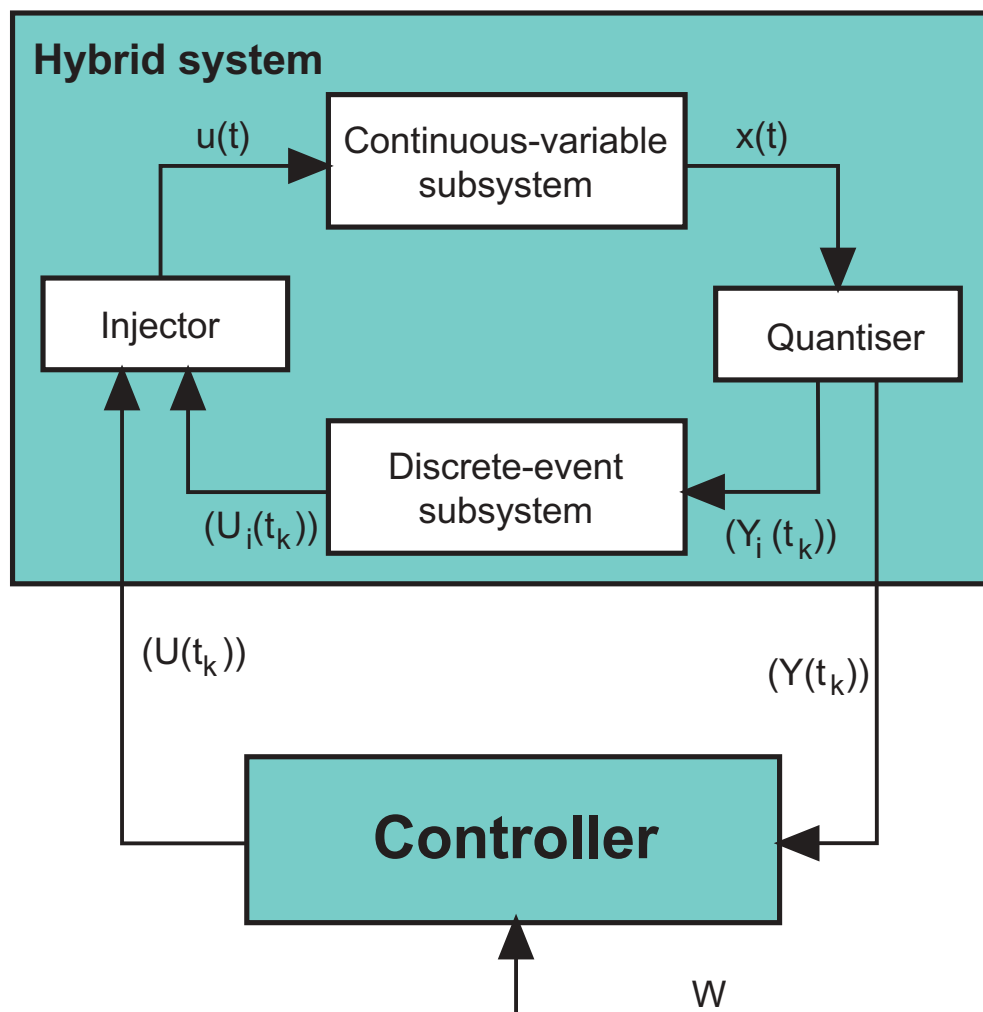
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1. Introduction to discrete-event modelling of hybrid systems
2. Properties of quantised systems
3. Some basics of automata theory
4. Discrete-event modelling of quantised systems by abstraction
5. Diagnosis of automata
6. Diagnosis of quantised systems
7. Application examples
8. Conclusions

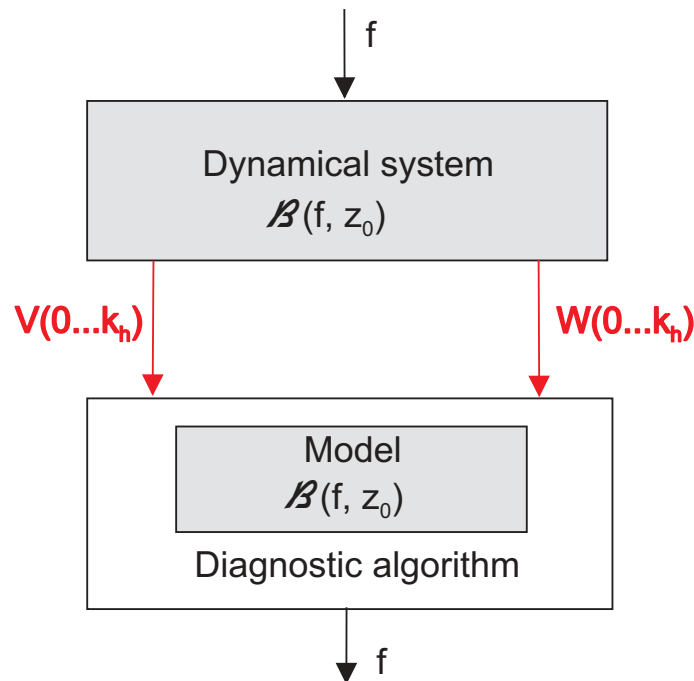
Supervisory control loop



Ways for dealing with hybrid systems

- Combine methods elaborated in continuous and discrete-event systems theories
- Abstract a discrete-event representation of the hybrid system and apply discrete-event systems theory

Model-based diagnosis



Diagnostic problem

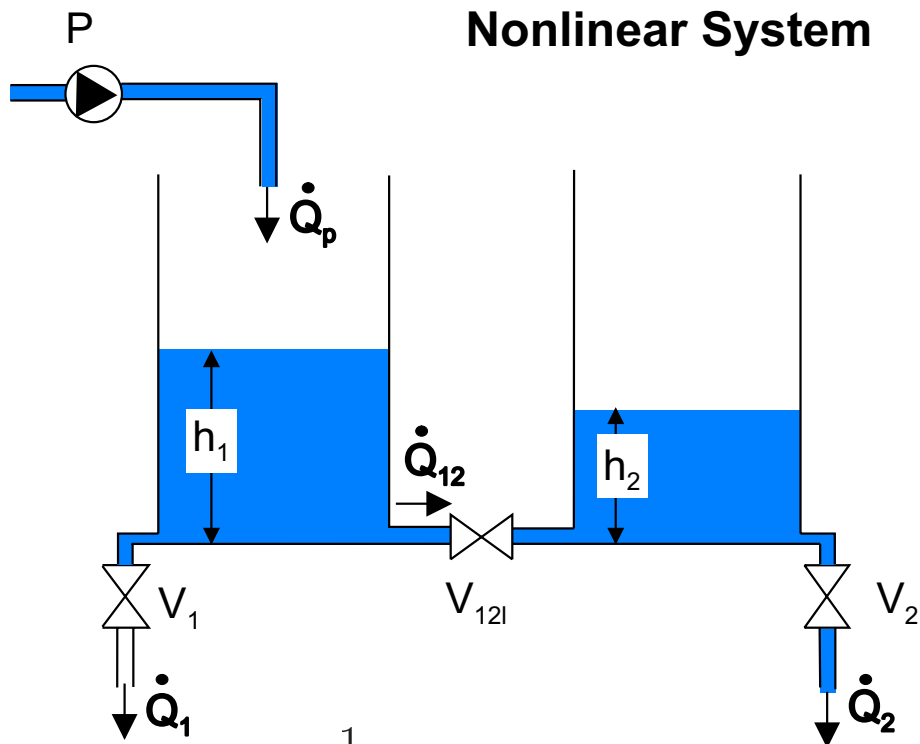
Given: Model depending on f and z_0
Measured I/O pair (V, W)

Consistency-based diagnosis:

Can the system subject to fault f generate the output W if it obtains the input V ?

- Diagnostic problems include **observation** problems.
- **Fault detection:**
Inconsistency with the faultless system
- **Fault identification:**
Consistency with the system subject to fault f
 $\rightarrow f$ is a fault candidate

Example: A batch process



$$\dot{h}_1 = \frac{1}{A_1} (\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12})$$

$$\dot{h}_2 = \frac{1}{A_2} (\dot{Q}_{12} - \dot{Q}_2)$$

$$\dot{Q}_1 = Pos(V_1) S_v \sqrt{2gh_1}$$

$$\dot{Q}_{12} = Pos(V_{12l}) S_v \operatorname{sgn}(h_1 - h_2) \sqrt{2g|h_1 - h_2|}$$

$$\dot{Q}_2 = Pos(V_2) S_v \sqrt{2gh_2}$$

$$\dot{Q}_p = p(t) \dot{Q}_{p0}$$

with

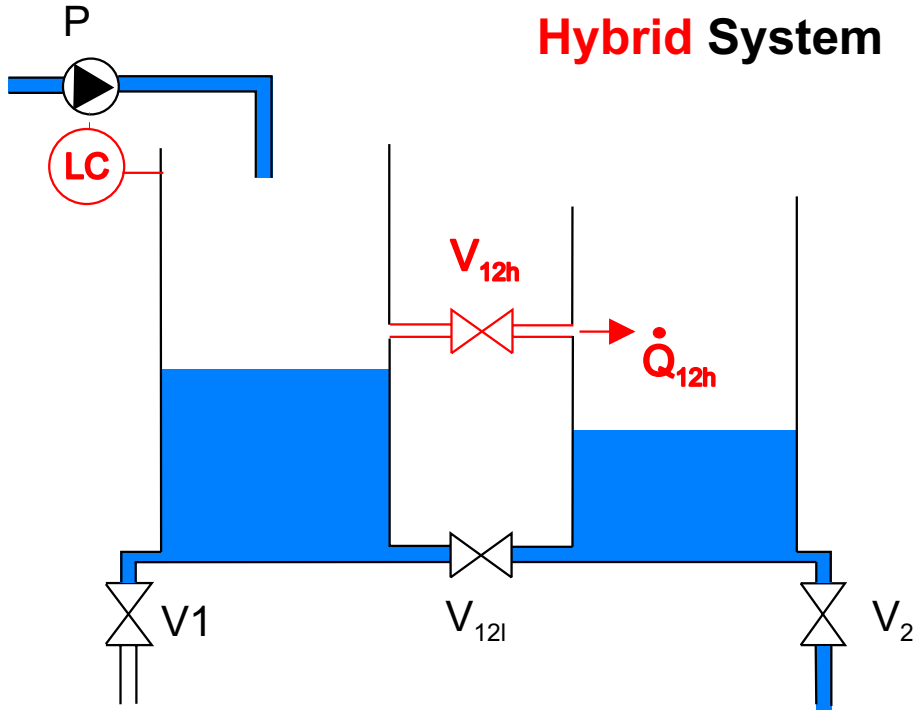
$$0 \leq h_1(t), h_2(t) \leq h_{max}$$

$$0 \leq p(t) \leq 1$$

\Downarrow

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t))$$



$$\dot{h}_1 = \frac{1}{A_1} (\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12} - \dot{Q}_{12h})$$

$$\dot{h}_2 = \frac{1}{A_2} (\dot{Q}_{12} + \dot{Q}_{12h} - \dot{Q}_2)$$

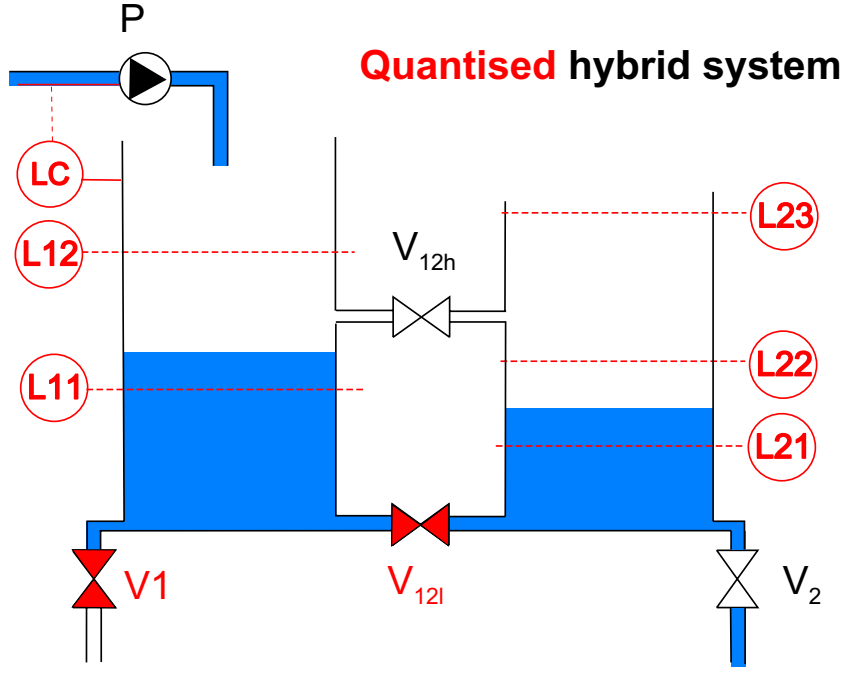
$$\dot{Q}_1 = \begin{cases} Pos(V_1) S_v \sqrt{2gh_1} & \text{if } h_1 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot{Q}_{12} = Pos(V_{12l}) S_v \operatorname{sgn}(h_1 - h_2) \sqrt{2g|h_1 - h_2|}$$

$$\dot{Q}_{12h} = \begin{cases} Pos(V_{12h}) S_v \operatorname{sgn}(h_1 - h_2) \sqrt{2g|h_1 - h_2|} & \text{if } h_1, h_2 > h_v \\ Pos(V_{12h}) S_v \sqrt{2g|h_1 - h_v|} & \text{if } h_1 > h_v, h_2 \leq h_v \\ -Pos(V_{12h}) S_v \sqrt{2g|h_2 - h_v|} & \text{if } h_2 > h_v, h_1 \leq h_v \\ 0 & \text{if } h_1, h_2 \leq h_v \end{cases}$$

$$\dot{Q}_2 = \begin{cases} Pos(V_2) S_v \sqrt{2gh_2} & \text{if } h_2 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot{Q}_p = \begin{cases} p(t) \dot{Q}_{p0} & \text{if } h_1 < h_{1max} \\ 0 & \text{if } h_1 \geq h_{1max} \end{cases}$$



$$\dot{h}_1 = \frac{1}{A_1} (\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12} - \dot{Q}_{12h})$$

$$\dot{h}_2 = \frac{1}{A_2} (\dot{Q}_{12} + \dot{Q}_{12h} - \dot{Q}_2)$$

$$\dot{Q}_1 = \begin{cases} Pos(V_1) S_v \sqrt{2gh_1} & \text{if } h_1 > 0 \\ 0 & \text{else} \end{cases}$$

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$$\dot{Q}_2 = \begin{cases} Pos(V_2) S_v \sqrt{2gh_2} & \text{if } h_2 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot{Q}_p = 0 \quad \text{if } h_1 \geq h_{1max}$$

$$\dot{Q}_p \in [0.8 \dot{Q}_{p0}, \dot{Q}_{p0}] \quad \text{if } h_1 < h_{1max}$$

$$Pos(V_1(t)), Pos(V_{12l}(t)) \in \{0, 1\}$$

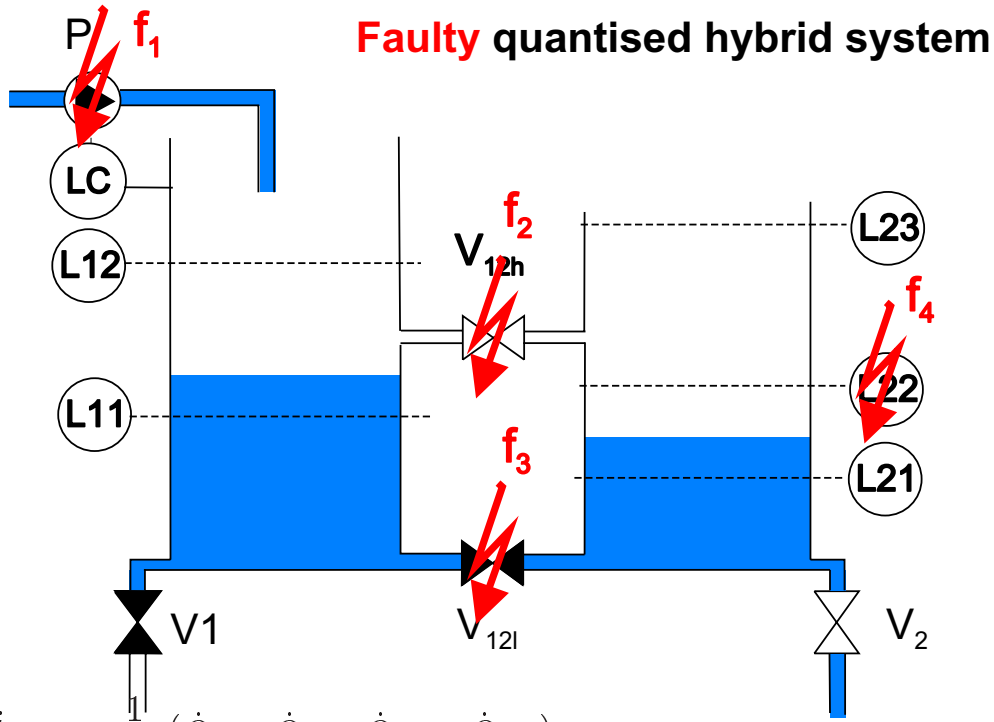
$$L_{11}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{11} \\ 0 & \text{else} \end{cases}$$

$$L_{12}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{12} \\ 0 & \text{else} \end{cases}$$

$$L_{21}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{21} \\ 0 & \text{else} \end{cases}$$

$$L_{22}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{22} \\ 0 & \text{else} \end{cases}$$

$$L_{23}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{23} \\ 0 & \text{else} \end{cases}$$



$$\dot{h}_1 = \frac{1}{A_1} (\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12} - \dot{Q}_{12h})$$

$$\dot{h}_2 = \frac{1}{A_2} (\dot{Q}_{12} + \dot{Q}_{12h} - \dot{Q}_2)$$

$$\dot{Q}_1 = \begin{cases} Pos(V_1) S_v \sqrt{2gh_1} & \text{if } h_1 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot{Q}_{12} = (1 - f_3) Pos(V_{12l}) S_v \operatorname{sgn}(h_1 - h_2) \sqrt{2g|h_1 - h_2|}$$

$$\dot{Q}_{12h} = \begin{cases} (1 - f_2) Pos(V_{12h}) S_v \operatorname{sgn}(h_1 - h_2) \sqrt{2g|h_1 - h_2|} & \text{if } h_1, h_2 > h_v \\ (1 - f_2) Pos(V_{12h}) S_v \sqrt{2g|h_1 - h_v|} & \text{if } h_1 > h_v, h_2 \leq h_v \\ -(1 - f_2) Pos(V_{12h}) S_v \sqrt{2g|h_2 - h_v|} & \text{if } h_2 > h_v, h_1 \leq h_v \\ 0 & \text{if } h_1, h_2 \leq h_v \end{cases}$$

$$\dot{Q}_2 = \begin{cases} Pos(V_2) S_v \sqrt{2gh_2} & \text{if } h_2 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot{Q}_p = \begin{cases} (1 - f_1) p(t) \dot{Q}_{p0} & \text{if } h_1 < h_{1max} \\ 0 & \text{if } h_1 \geq h_{1max} \end{cases}$$

$Pos(V_1(t)), Pos(V_{12l}(t)) \in \{0, 1\}$

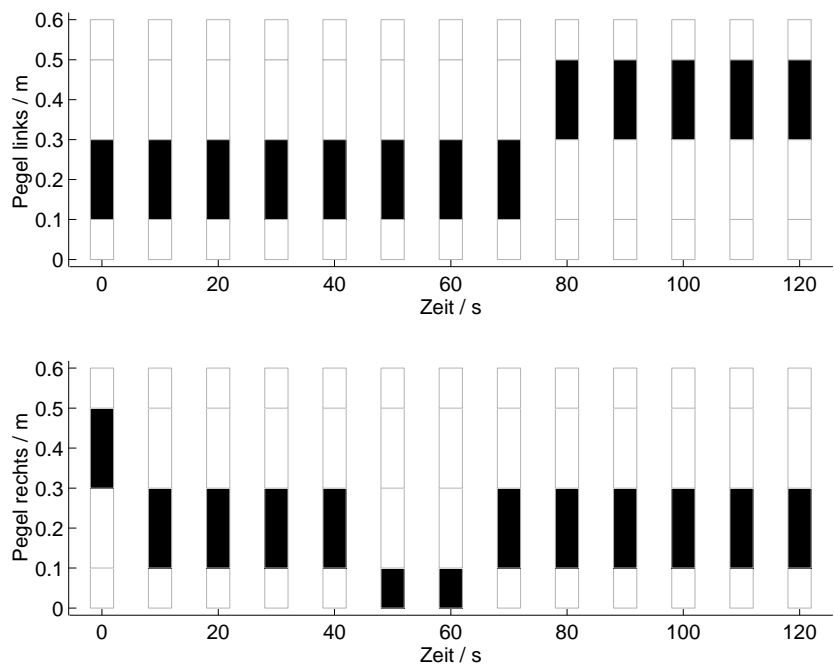
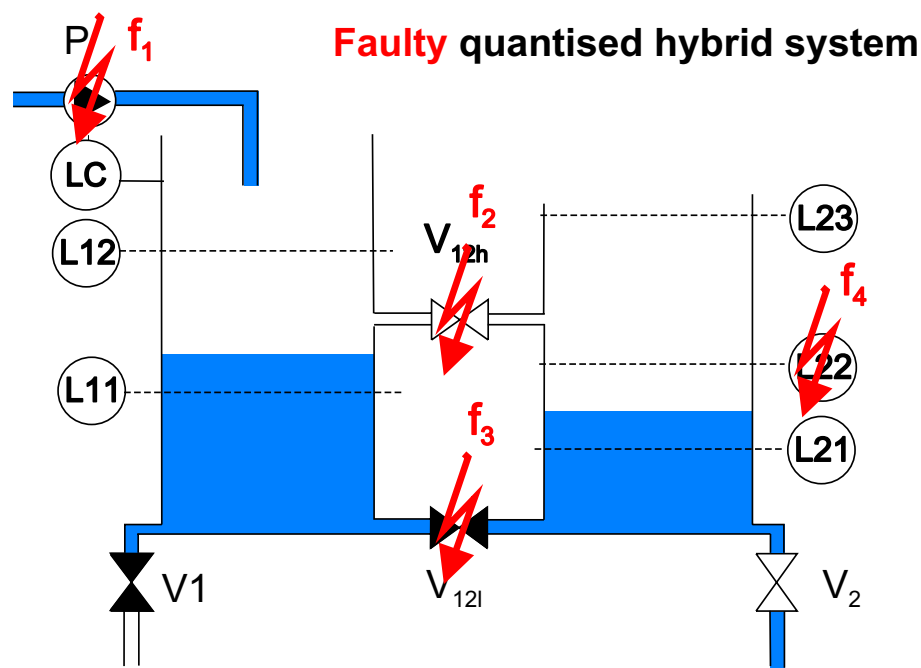
$$f_1, f_2, f_3, f_4 \in \{0, 1\}$$

$$L_{11}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{11} \\ 0 & \text{else} \end{cases} \quad L_{12}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{12} \\ 0 & \text{else} \end{cases}$$

$$L_{21}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{21} \\ 0 & \text{else} \end{cases} \quad L_{22}(t) = \begin{cases} (1 - f_4) & \text{if } h_1(t) > l_{22} \\ 0 & \text{else} \end{cases}$$

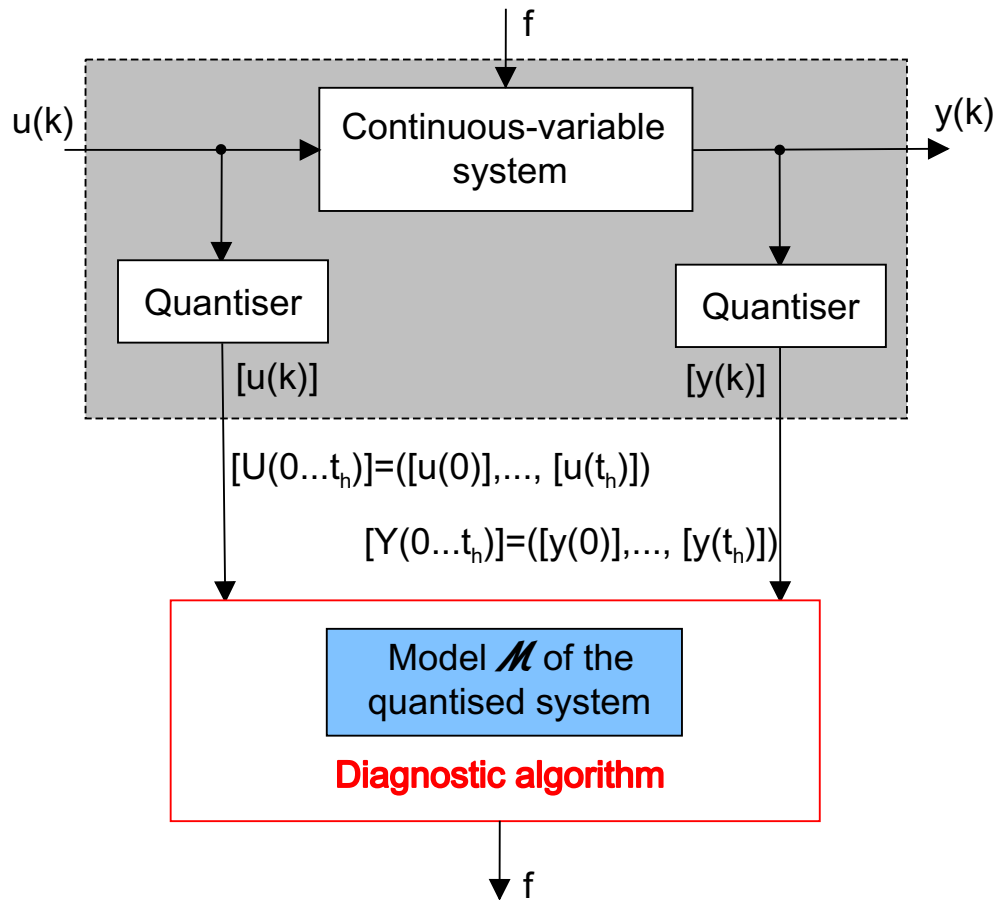
$$L_{23}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{23} \\ 0 & \text{else} \end{cases}$$

Diagnosis of the tank system



Is the tank system faulty?

Diagnosis of quantised systems



Solution steps

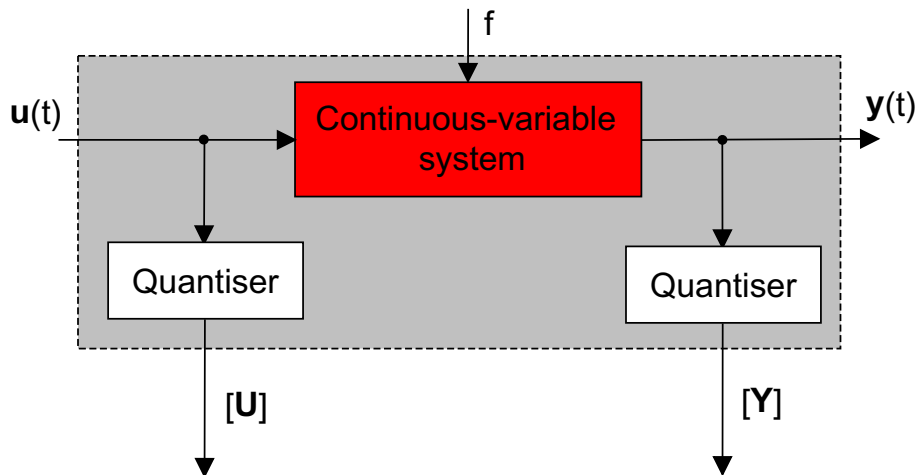
1. Modelling

Find a discrete-event representation of the quantised system

2. Diagnosis

Find a method to decide whether the quantised system behaves like the discrete-event model

Quantised systems



Continuous-variable system:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k)), & \mathbf{x}(0) &= \mathbf{x}_0 & (*) \\ \mathbf{y}(k) &= \mathbf{g}(\mathbf{x}(k), \mathbf{u}(k)) & & & (**) \end{aligned}$$

For given \mathbf{x}_0 and

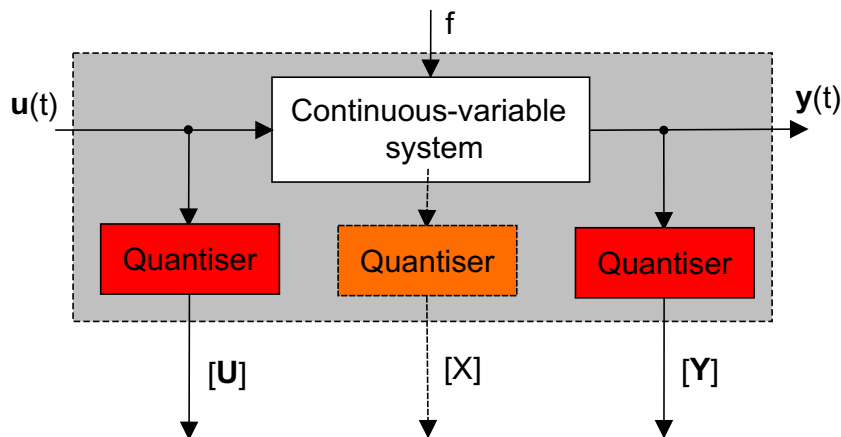
$$\mathbf{U}(0...t_h) = (\mathbf{u}(0), \mathbf{u}(1), \mathbf{u}(2), \dots, \mathbf{u}(t_h))$$

the system generates

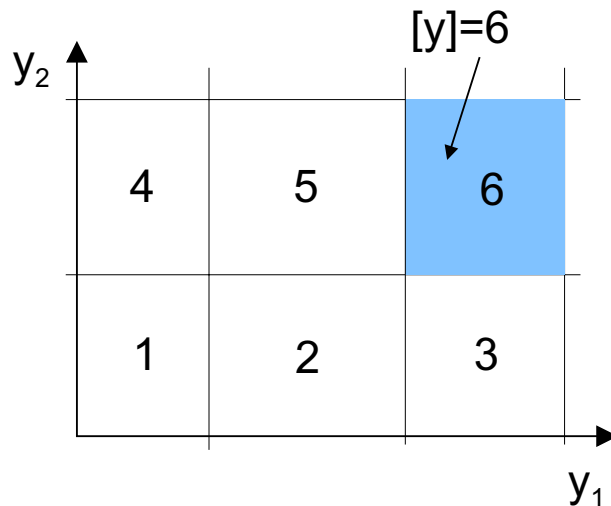
$$\mathbf{X}(0...t_h) = (\mathbf{x}(0), \mathbf{x}(1), \mathbf{x}(2), \dots, \mathbf{x}(t_h))$$

$$\mathbf{Y}(0...t_h) = (\mathbf{y}(0), \mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(t_h))$$

Quantised systems



Output quantisation:



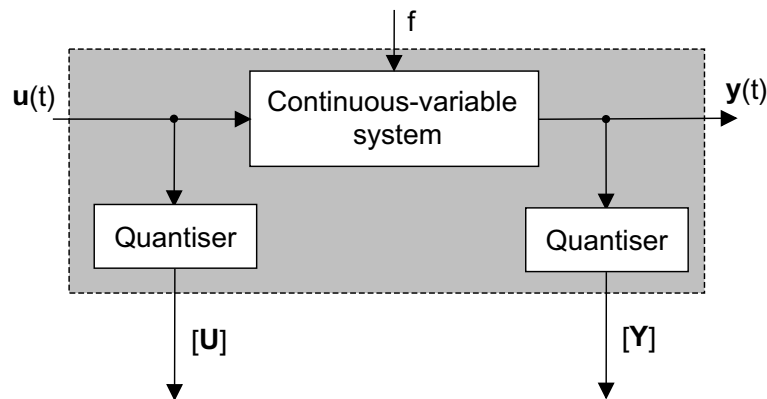
$$[\mathbf{y}(k)] = w \quad \text{if} \quad \mathbf{y}(k) \in \mathcal{Q}_y(w) \quad \mathcal{N}_w = \{0, 1, 2, \dots, R\}$$

Input and state quantisation

$$[\mathbf{u}(k)] = v \quad \text{if} \quad \mathbf{u}(k) \in \mathcal{Q}_u(v) \quad \mathcal{N}_v = \{0, 1, 2, \dots, M\}$$

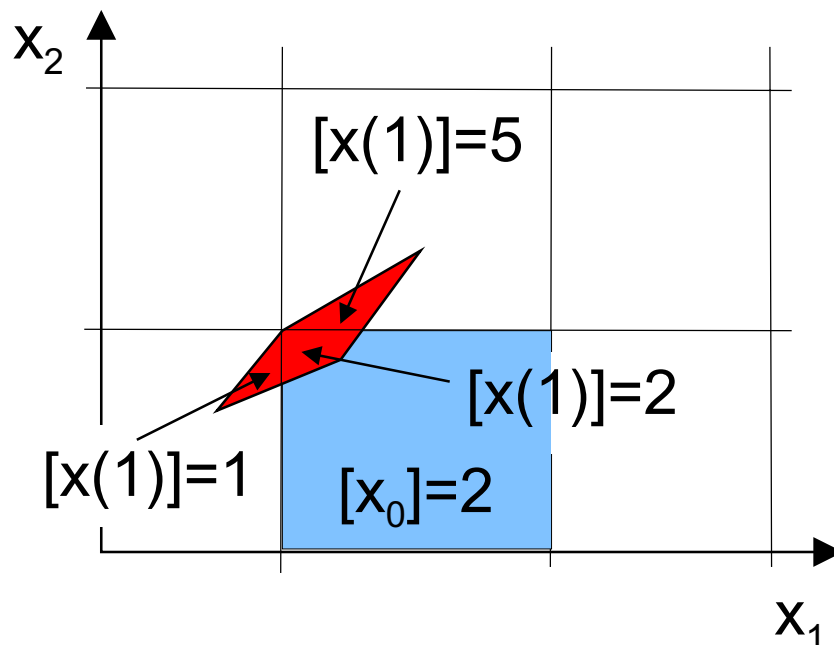
$$[\mathbf{x}(k)] = z \quad \text{if} \quad \mathbf{x}(k) \in \mathcal{Q}_x(z) \quad \mathcal{N}_z = \{0, 1, 2, \dots, N\}$$

Nondeterminism of the quantised system behaviour



For given quantised initial state $[x(0)]$ and quantised input $[u(0)]$ the system may generate more than one quantised successor state $[x(1)]$

$$x(1) = f(x(0), u(0)),$$

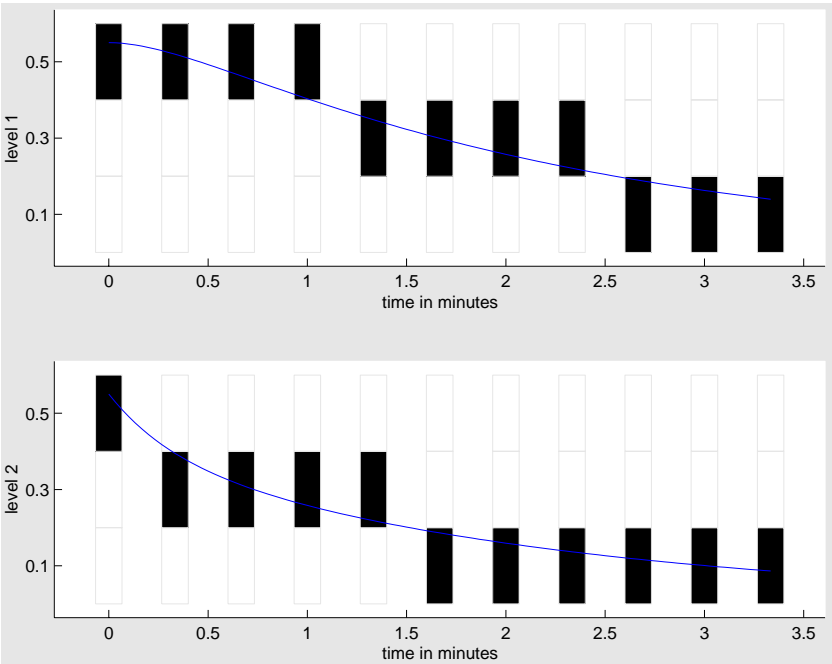
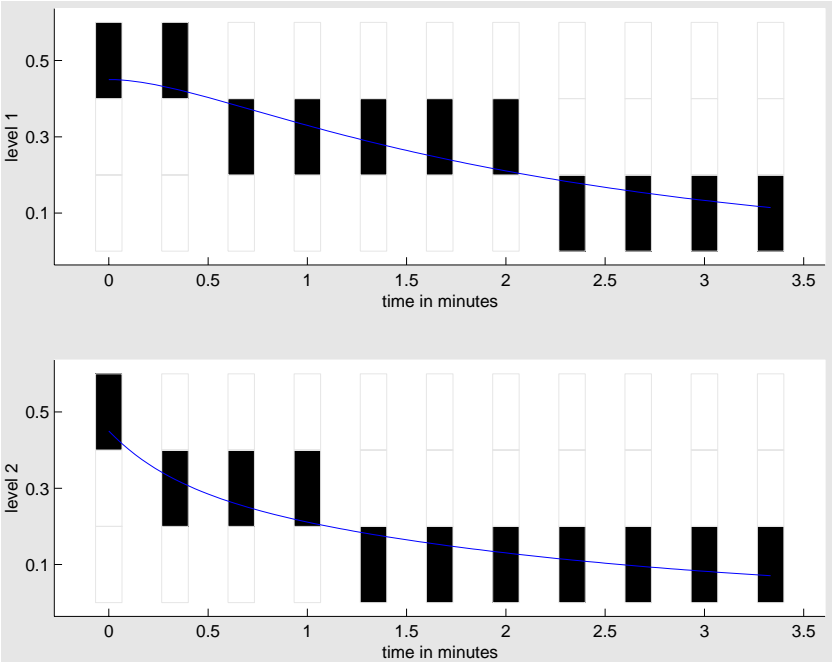


Consequence:

Discrete-event models of quantised systems have to be nondeterministic.

Nondeterminism of the quantised system behaviour

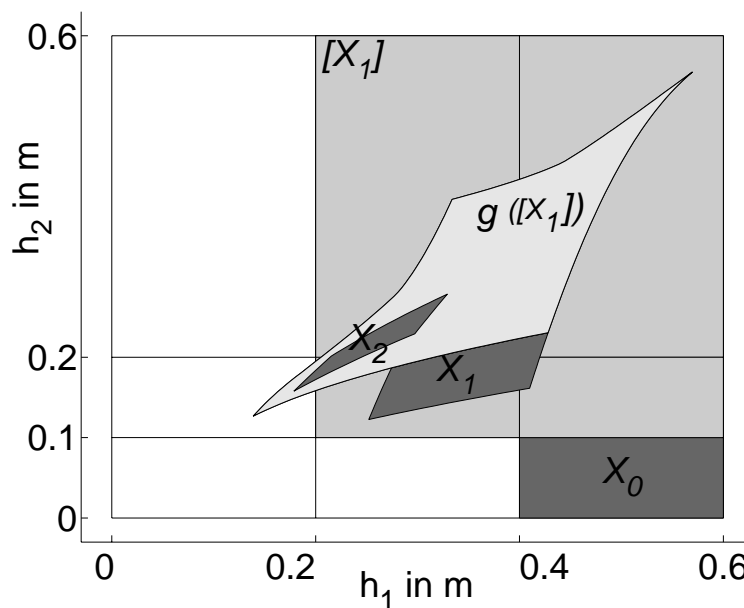
Behaviour of the quantised tank system



Nondeterminism of the quantised system behaviour

Quantised systems do **not** possess the Markov property

$$\begin{aligned} \text{Prob}([\mathbf{x}(k+1)] \mid [\mathbf{x}(k)], [\mathbf{x}(k-1)], \dots, [\mathbf{x}(0)]) \\ = \text{Prob}([\mathbf{x}(k+1)] \mid [\mathbf{x}(k)]) \end{aligned}$$



Exception: (Lunze 1994)

- linear autonomous system $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k)$
- equidistant partitioning with resolution q_{xi}
- $\mathbf{A} = \text{diag } q_{xi} \text{ diag } (2n_i + 1)^{-1} \mathbf{P} \text{ diag } q_{xi}^{-1}$

Consequence:

No representation form, which possesses the Markov property, can precisely describe a quantised system

Modelling problem

Given: Quantised system

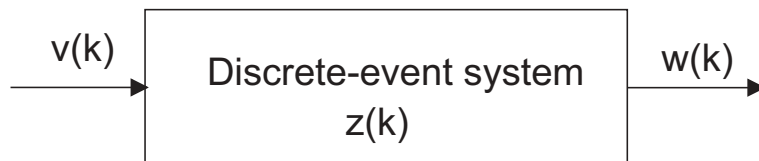
Find: Automaton with the following property:

Set of model trajectories \supseteq Set of system trajectories

$$\mathbf{Z}([\mathbf{x}(0)], [\mathbf{U}]) \supseteq [\tilde{\mathbf{X}}([\mathbf{x}(0)], [\mathbf{U}])]$$

- Such a model is called **complete**.
- Spurious solutions = Model trajectories that the quantised system cannot follow

Basics of automata theory



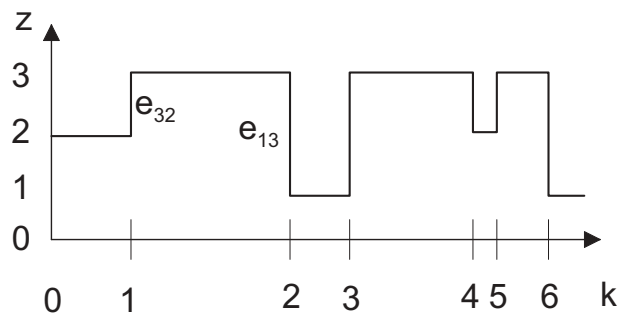
Discrete signal spaces

$$v \in \mathcal{N}_v = \{1, 2, \dots, M\}$$

$$z \in \mathcal{N}_z = \{1, 2, \dots, N\}$$

$$w \in \mathcal{N}_w = \{1, 2, \dots, R\}$$

Event = change of the input, state or output



State sequence

$$Z(0\dots 6) = (2, 3, 1, 3, 2, 3, 1)$$

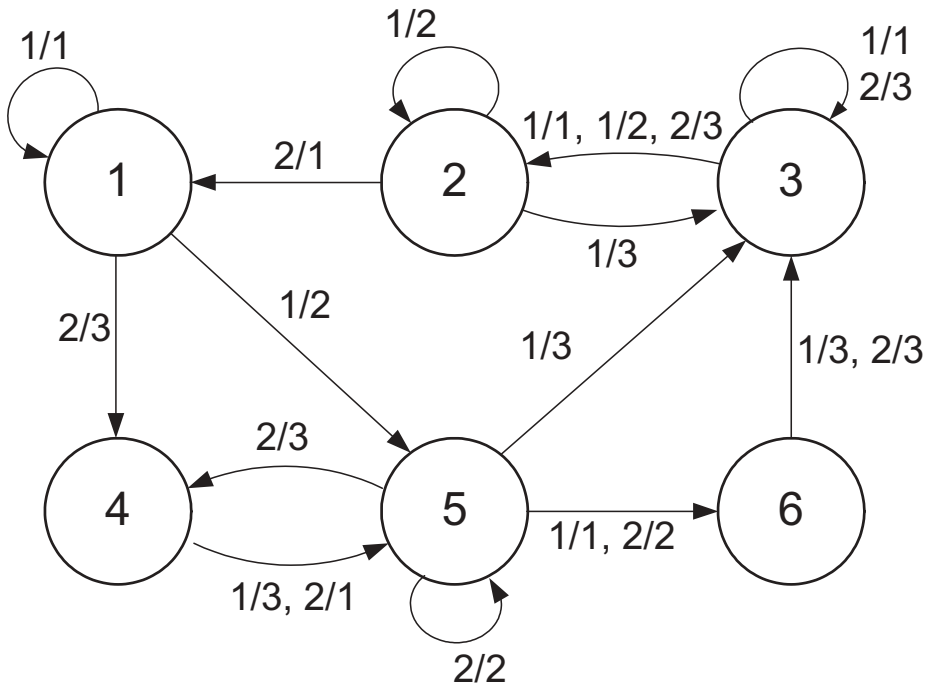
Nondeterministic automaton

$$\mathcal{N}(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, \textcolor{red}{L}, z(0))$$

State transition relation

$$\textcolor{red}{L} : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \rightarrow \{0, 1\}$$

$L(z', w, z, v) = 1 \Rightarrow$ automaton may jump
from $z(k) = z$ towards $z(k+1) = z'$
while producing the output $w(k) = w$ for input $v(k) = v$



Stochastic automaton

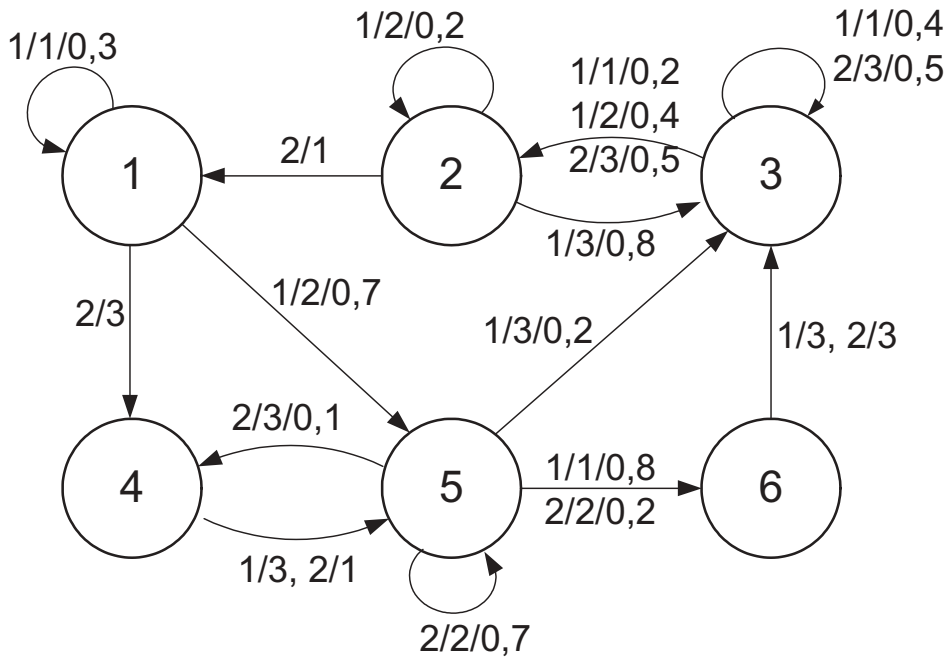
$$\mathcal{S}(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, \mathbf{L}, \text{Prob}(z(0)))$$

State transition probability distribution

$$L : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \longrightarrow [0, 1]$$

$$\mathbf{L}(z', w \mid z, v) =$$

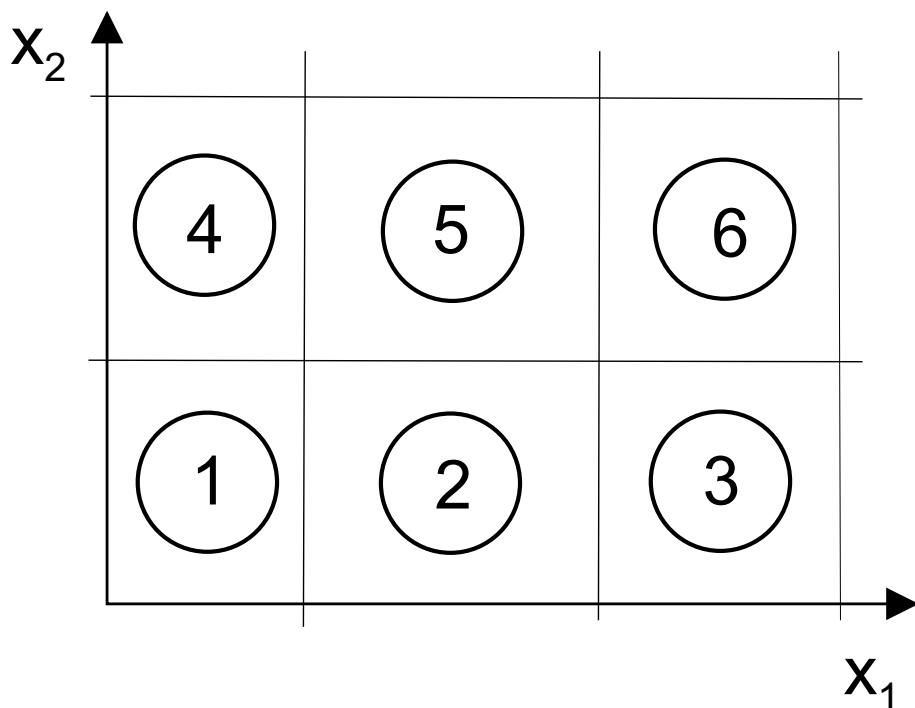
$$\text{Prob}(z_p(1) = z', w_p(0) = w \mid z_p(0) = z, v_p(0) = v)$$



Modelling of quantised systems by stochastic automata

Stochastic automaton $\mathcal{S}(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L, \text{Prob}(z_0))$

- \mathcal{N}_z - set of quantised state symbols
- \mathcal{N}_v - set of quantised input symbols
- \mathcal{N}_w - set of quantised output symbols



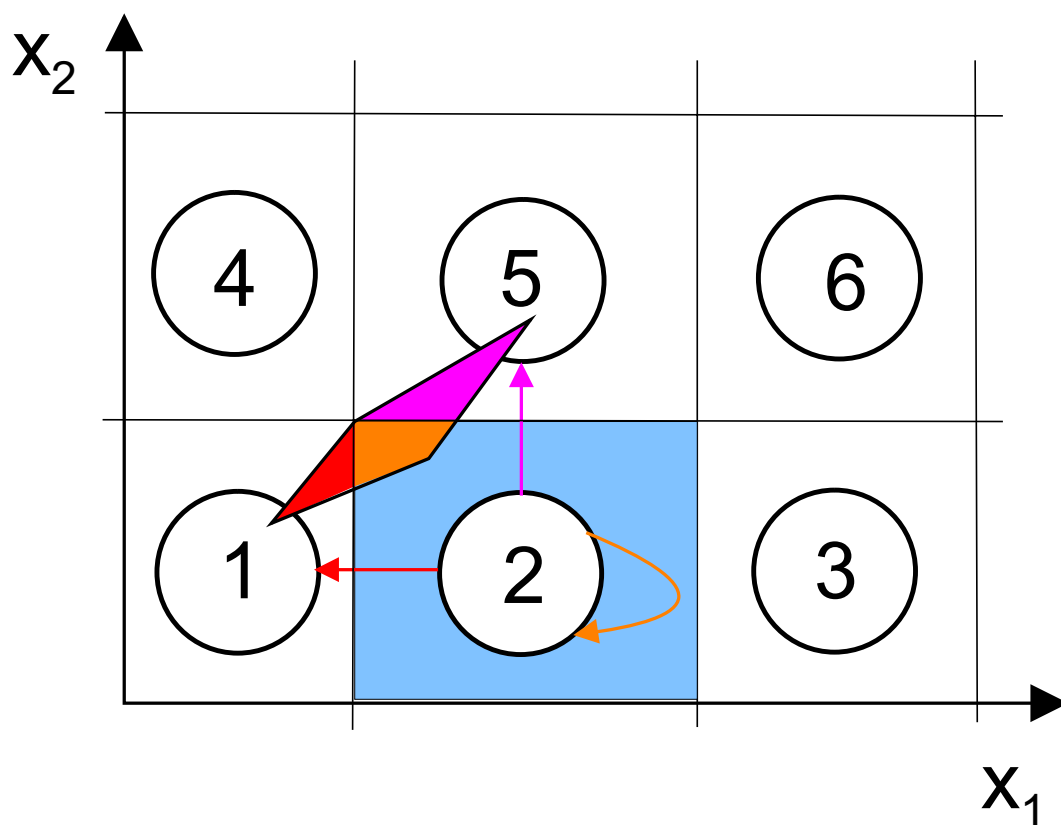
Modelling of quantised systems by stochastic automata

Abstraction

$$L(z', w \mid z, v) =$$

$$\text{Prob}([\mathbf{x}(1)] = z', [\mathbf{y}(0)] = w \mid [\mathbf{x}(0)] = z, [\mathbf{u}(0)] = v)$$

$$\text{Prob}(z_0) > 0 \text{ for } z_0 = [\mathbf{x}_0]$$

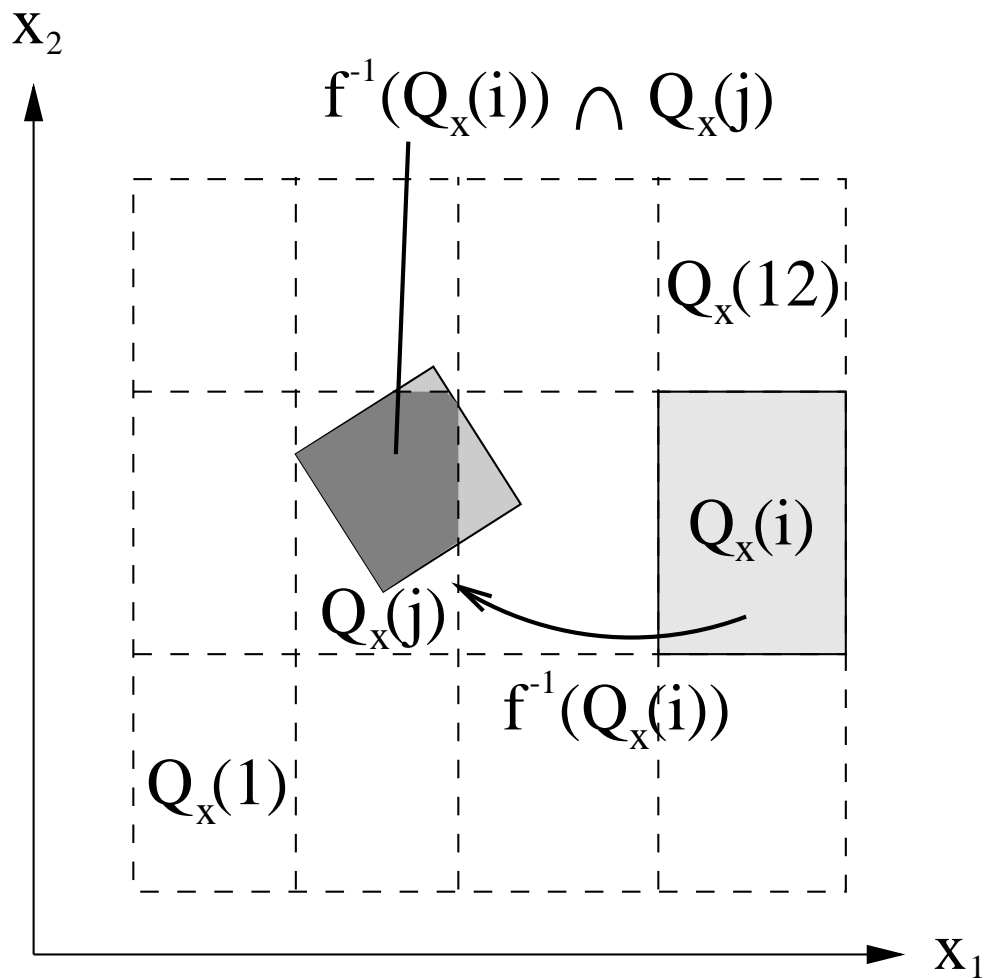


Modelling of quantised systems by stochastic automata

Abstraction

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k))$$

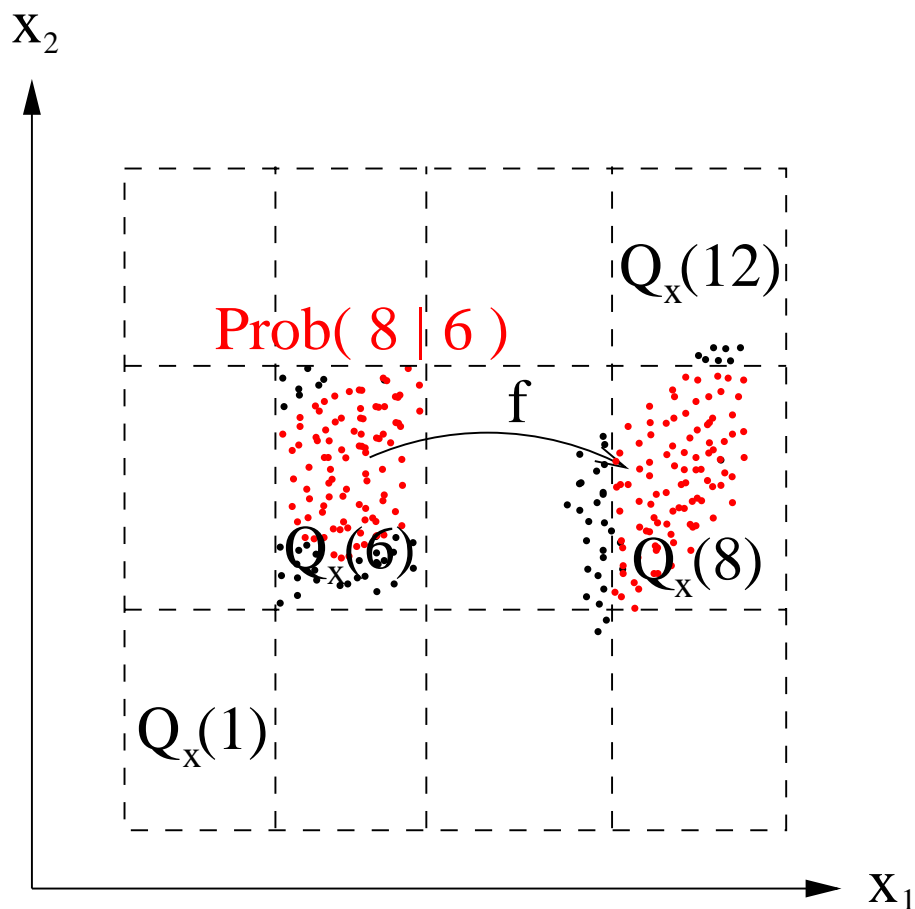
$$L(i \mid j) = \frac{\lambda(\boldsymbol{f}^{-1}(\mathcal{Q}_x(i)) \cap \mathcal{Q}_x(j))}{\lambda(\mathcal{Q}_x(j))}$$



Modelling of quantised systems by stochastic automata

Abstraction

Point-based cell-to-cell mapping



The automaton obtained is, in general, **incomplete**.

Modelling of quantised systems by stochastic automata

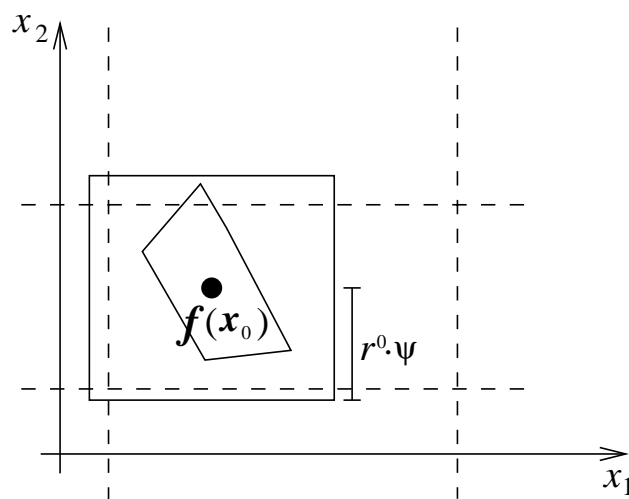
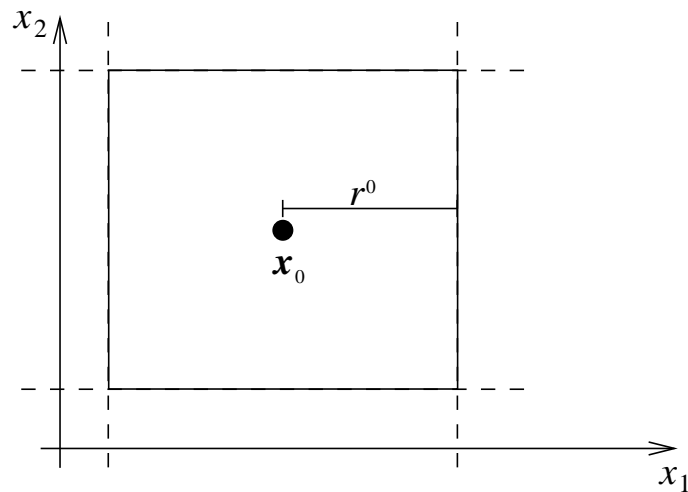
Hyperbox mapping

(Lunze, Schröder, ECC 2001)

Assumption:

System satisfies a Lipschitz condition:

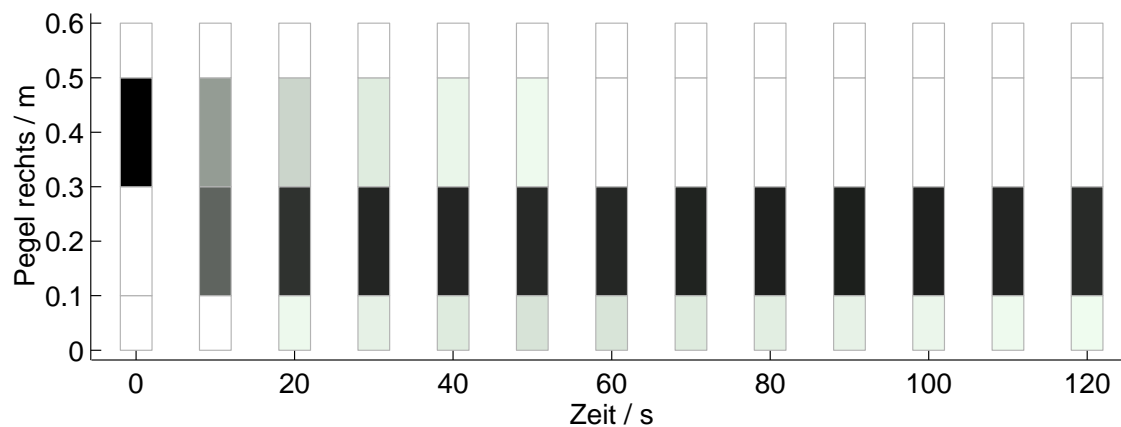
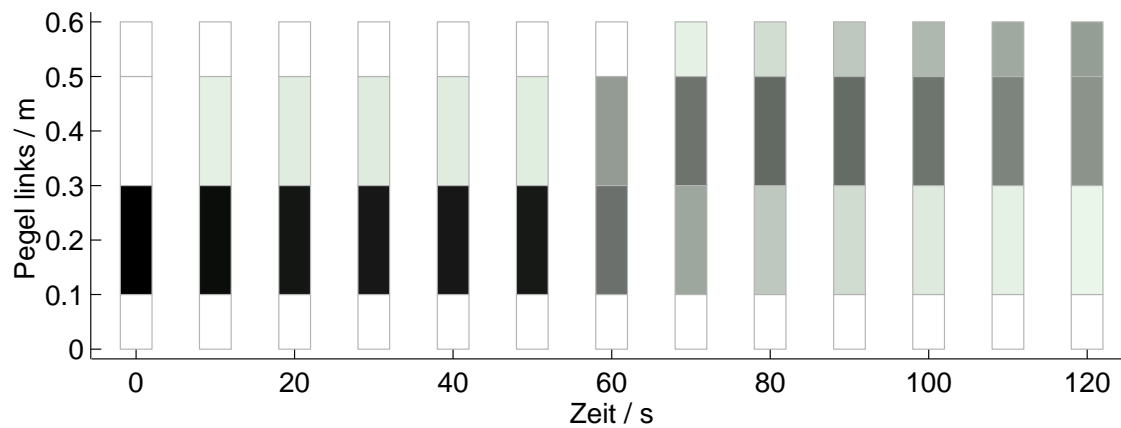
$$\|\mathbf{f}(\mathbf{x}_1) - \mathbf{f}(\mathbf{x}_2)\|_\infty \leq \psi \cdot \|\mathbf{x}_1 - \mathbf{x}_2\|_\infty$$



The automaton is complete.

Modelling of quantised systems by stochastic automata

Simulation:



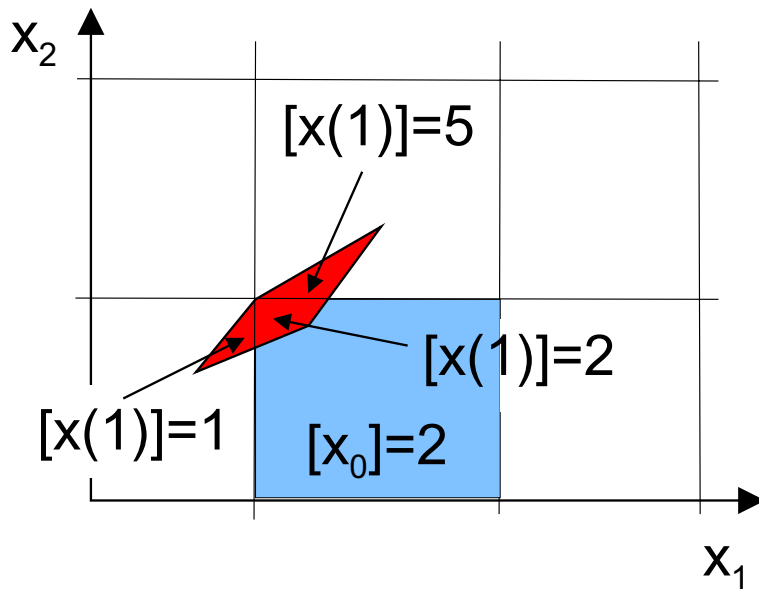
The stochastic automaton is a complete model.

The state partitioning problem

to appear in Blondel/Megretski: 60 open problems..., Princeton Univ. Press
2003

Under what conditions is the discrete-event behaviour deterministic?

$$\forall i \in \mathcal{N}_z \quad \exists j : \mathbf{f}(\mathbf{x}) \in \mathcal{Q}(j) \text{ for all } \mathbf{x} \in \mathcal{Q}(i)$$



Problem A

Given: $\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k))$

Partition \mathcal{Q}_x

Find: Conditions on \mathbf{f} , \mathcal{Q}_x such that
the discrete-event behaviour is deterministic

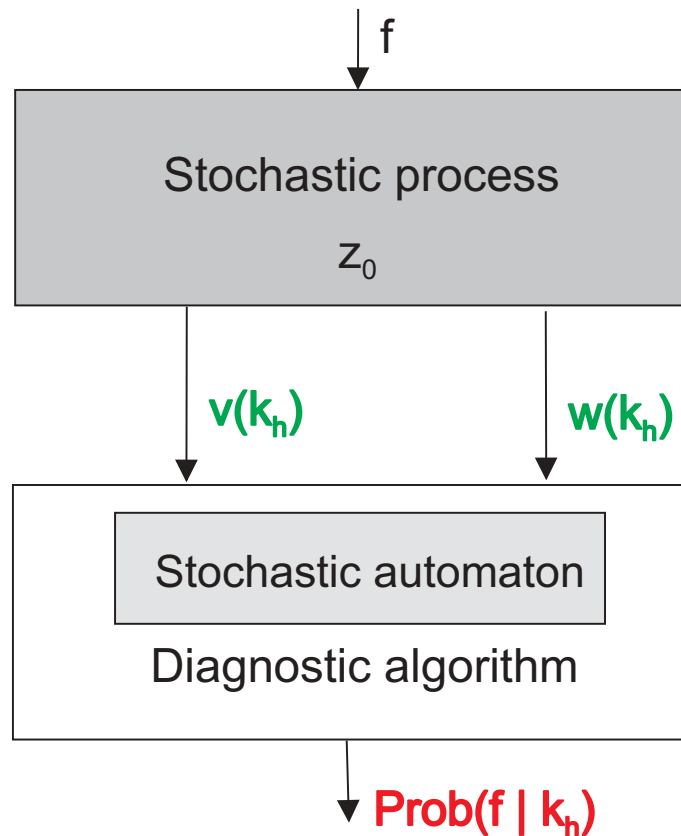
Problem B

Given: $\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k))$

Find: Partition \mathcal{Q}_x such that
the discrete-event behaviour is deterministic

Diagnosis of automata

Assumption: The fault f is time-invariant



Diagnostic problem

Given: Stochastic automaton \mathcal{S}
 $V(0...k_h), W(0...k_h)$

Find: Fault f

Restriction to the fault detection problem: test the consistency with the model of the faultless system

State observation of nondeterministic automata

Consistency check:

Is the I/O pair consistent with the automaton?

Given: I/O pair

$$V(0...k_h) = (v(0), v(1), \dots, v(k_h))$$

$$W(0...k_h) = (w(0), w(1), \dots, w(k_h))$$

The I/O pair is consistent with the automaton if there exists a state sequence $Z(0...k_h)$ such that

$$L(z(k+1), w(k), z(k), v(k)) = 1 \quad \text{for all } k$$

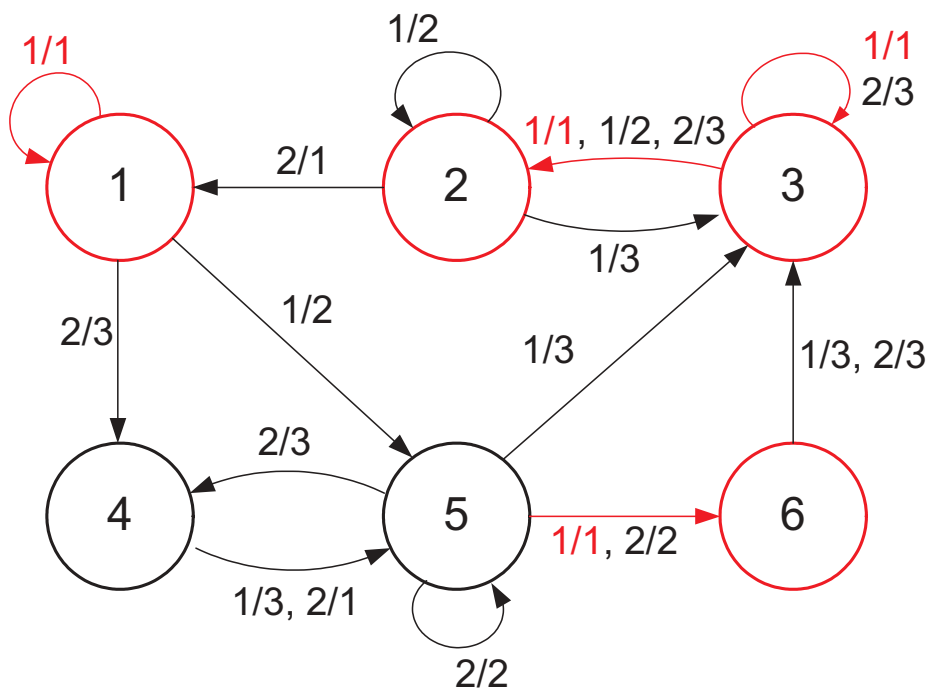
The consistency check includes a state observation problem.

State observation of nondeterministic automata

A-priori information:

$$\mathcal{Z}(0 \mid -1) = \{1, 2, 3, 4, 5, 6\}$$

Measurement: $v(0) = 1, w(0) = 1$



Information about the state obtained by the measurement:

$$\mathcal{Z}(0 \mid 0) = \{1, 3, 5\}$$

$$\mathcal{Z}(1 \mid 0) = \{\textcolor{red}{1}, \textcolor{red}{2}, \textcolor{red}{3}, \textcolor{red}{6}\}$$

State observation of nondeterministic automata

$$\mathcal{Z}(k_h \mid k_h) = \{z \in \mathcal{Z}(k_h \mid k_h - 1) : \exists z' : L(z', w, z, v) = 1\}$$

$$\begin{aligned} \mathcal{Z}(k_h + 1 \mid k_h) = \{z' : \exists z : L(z', w, z, v) = 1 \\ \text{for a } z \in \mathcal{Z}(k_h \mid k_h)\} \end{aligned}$$

Consistency check:

The I/O pair is consistent with the automaton
if and only if

$$\mathcal{Z}(k_h \mid k_h) \neq \emptyset \quad \text{for all } k_h$$

Algorithm *Observation of non-deterministic automata*

Given: Non-deterministic automaton \mathcal{N}

Initial state set $\mathcal{Z}(0 \mid -1)$

Init.: $\mathcal{Z}_r = \mathcal{Z}(0 \mid -1)$

$k_h = 0$

Do: 1. Measure the I/O pair (v, w)

2. Determine

$\mathcal{Z}_k := \{z \in \mathcal{Z}_r : L(z', w, z, v) = 1 \text{ for a } z' \in \mathcal{N}_z\}$

3. **Consistency check:**

If $\mathcal{Z}_k = \emptyset$, stop the algorithm

(inconsistent I/O pair or wrong initial state set)

4. Determine

$\mathcal{Z}_r := \{z' : L(z', w, z, v) = 1 \text{ for a } z \in \mathcal{Z}_k\}$

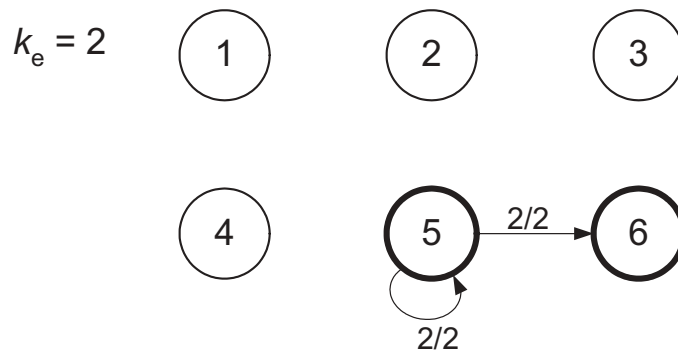
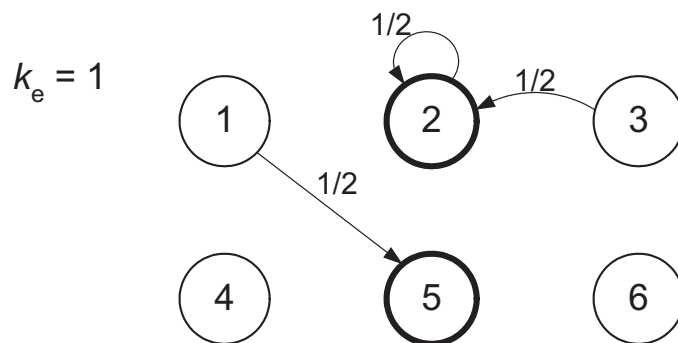
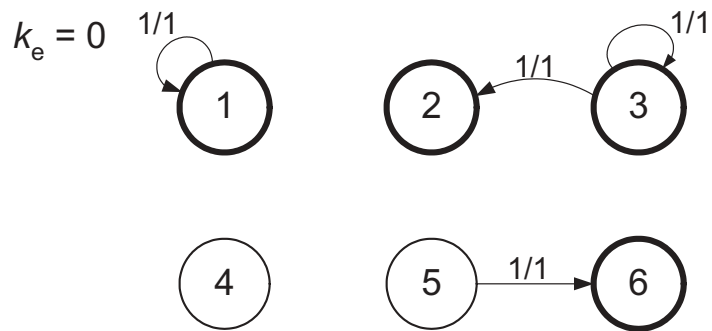
5. $k_h := k_h + 1$

Continue with Step 1

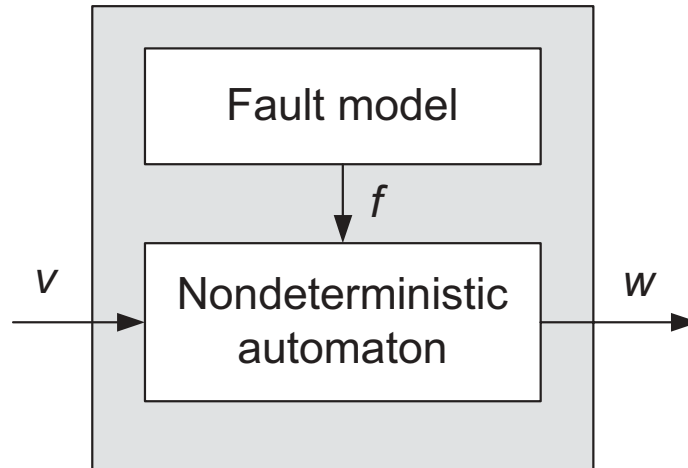
Result: $\mathcal{Z}_k = \mathcal{Z}(k_h \mid k_h)$ for increasing time horizon k_h

State observation of nondeterministic automata

I/O pair: $(V, W) = ((1, 1, 2, 2), (1, 2, 2, 3))$



Diagnosis of nondeterministic automata



Fault model: $\mathcal{N}_F(\mathcal{N}_f, G_F, z_{F0})$

Nondeterministic automaton including the fault model:

State:

$$\tilde{z} = \begin{pmatrix} z \\ f \end{pmatrix}$$

State transition relation:

$$\tilde{L} \left(\begin{pmatrix} z' \\ f' \end{pmatrix}, w, \begin{pmatrix} z \\ f \end{pmatrix}, v \right) = L(z', w, z, v, f) \cdot G(f', f),$$

Diagnosis is based on a consistency check on the I/O pair for this automaton.

Algorithm *Diagnosis of non-deterministic automata*

Given: Non-deterministic automaton \mathcal{N}

Fault model \mathcal{N}_F

Initial state set $\mathcal{Z}(0 \mid -1)$

Initial fault set $\mathcal{F}(0 \mid -1)$

Init.: $\mathcal{Z}_r = \mathcal{Z}(0 \mid -1) \times \mathcal{F}(0 \mid -1)$
 $k_h = 0$

Do: 1. Measure the I/O pair (v, w)

2. Determine

$$\tilde{\mathcal{Z}}_k := \{(z, f) \in \tilde{\mathcal{Z}}_r :$$

$$L(\tilde{z}', w, z, v, f) \cdot G(f', f) = 1 \text{ for a } \tilde{z}' \in \mathcal{N}_z\}$$

3. **Consistency check:**

If $\mathcal{Z}_k = \emptyset$, stop the algorithm

(wrong initial state set or initial fault set)

4. Determine

$$\mathcal{Z}_r := \{(z', f') : L(z', w, z, v, f) \cdot G(f', f) = 1 \\ \text{for a } (z, f) \in \tilde{\mathcal{Z}}_k\}$$

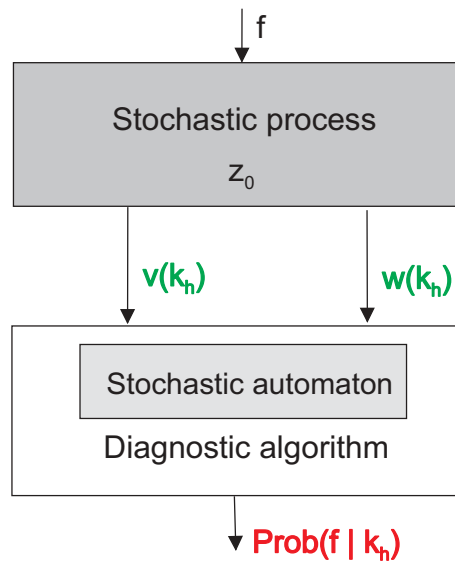
5. Determine $\mathcal{F}_k = \{f : (z, f) \in \tilde{\mathcal{Z}}_k\}$

6. $k_h := k_h + 1$

Continue with Step 1

Result: $\mathcal{F}(k_h \mid k_h)$ for increasing time horizon k_h

Diagnosis of stochastic automata



Diagnostic problem

Given: Stochastic automaton \mathcal{S}
 $V(0 \dots k_h), W(0 \dots k_h)$

Find: Fault f

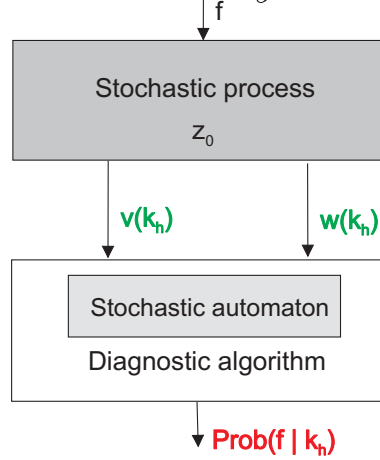
Do there exist some f, z_0 such that (V, W) is consistent with the automaton?

Result: $\text{Prob}(f \mid V(0 \dots k_h), W(0 \dots k_h)) =: \text{Prob}(f \mid k_h)$

$$\mathcal{F}(k_h) = \{f : \text{Prob}(f \mid k_h) > 0\}$$

Diagnosis of stochastic automata

(Lunze, Schröder, *Discrete Event Dynamic Systems*, 2001)



Initialisation: $\text{Prob}(f, z(0) \mid -1) = \text{Prob}(f, z(0))$

Iteration: $k_h := k_h + 1$

1. Measure $v(k_h), w(k_h)$

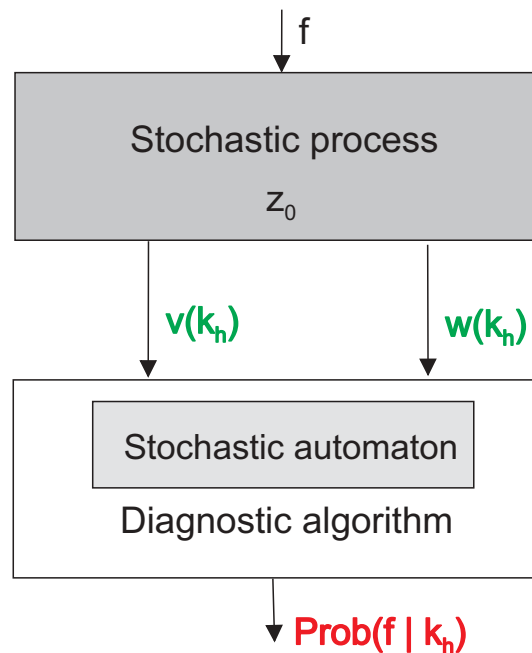
2. Determine $L(k_h) = L(z(k_h+1), w(k_h) \mid z(k_h), v(k_h), f)$

$$3. \text{Prob}(f \mid k_h) = \frac{\sum_{z(k_h)}^{z(k_h+1)} L(k_h) \cdot \text{Prob}(f, z(k_h) \mid k_h - 1)}{\sum_{z(k_h), f}^{z(k_h+1)} L(k_h) \cdot \text{Prob}(f, z(k_h) \mid k_h - 1)}$$

4. $\mathcal{F}(k_h) = \{f \mid \text{Prob}(f \mid k_h) > 0\}$

$$5. \text{Prob}(f, z(k_h + 1) \mid k_h) = \frac{\sum_{z(k_h)}^{z(k_h+1)} L(k_h) \cdot \text{Prob}(f, z(k_h) \mid k_h - 1)}{\sum_{z(k_h), f}^{z(k_h+1)} L(k_h) \cdot \text{Prob}(f, z(k_h) \mid k_h - 1)}$$

Diagnosis of stochastic automata



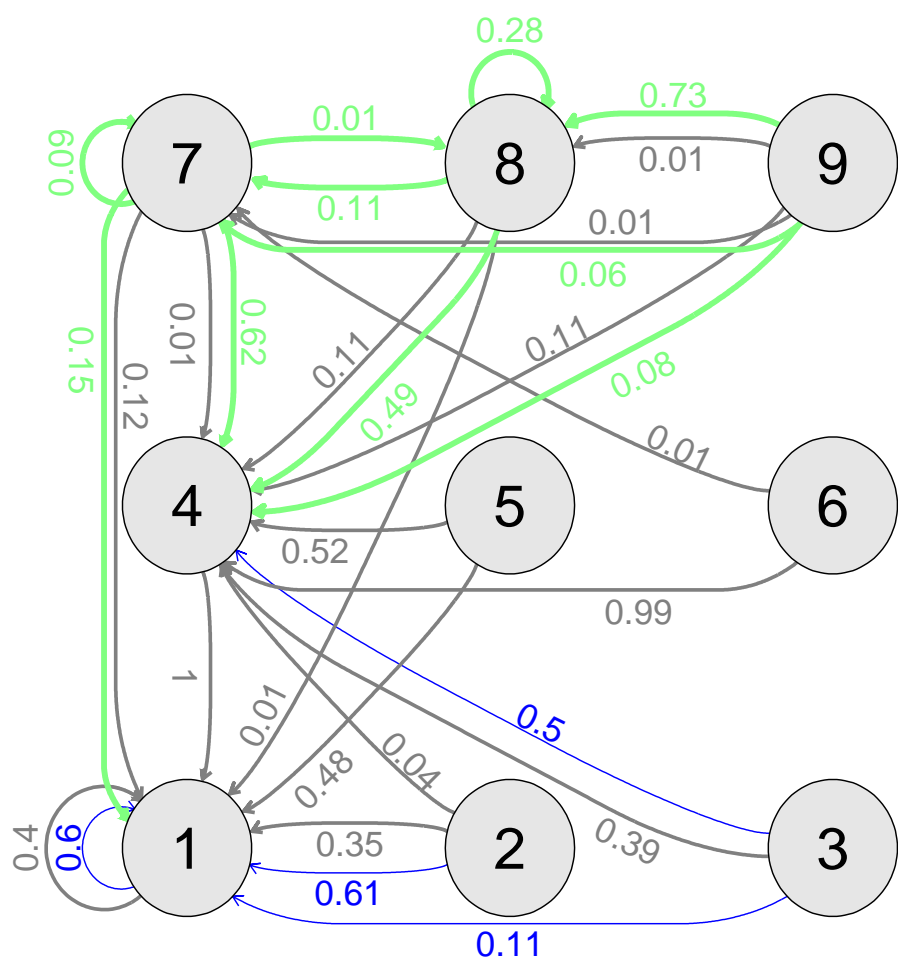
Result: $\text{Prob}(f | k_h)$

$$\mathcal{F}(k_h) = \{f : \text{Prob}(f | k_h) > 0\}$$

- The system is subject to some fault $f \in \mathcal{F}(k_h)$.
- **Fault detection:**
If $f_0 \notin \mathcal{F}(k_h)$ holds, the system is known to be subject to some fault.
- **Fault identification:**
If $\mathcal{F}(k_h) = \{f_i\}$ is a singleton, the system is known to be subject to fault f_i .

BRIDGE Benchmark problem

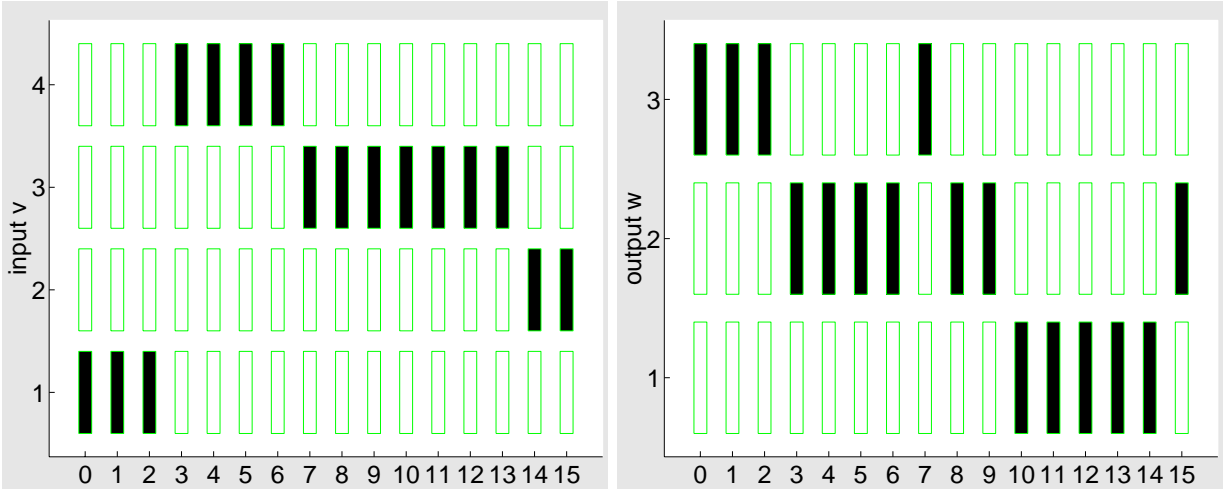
Scenario 2



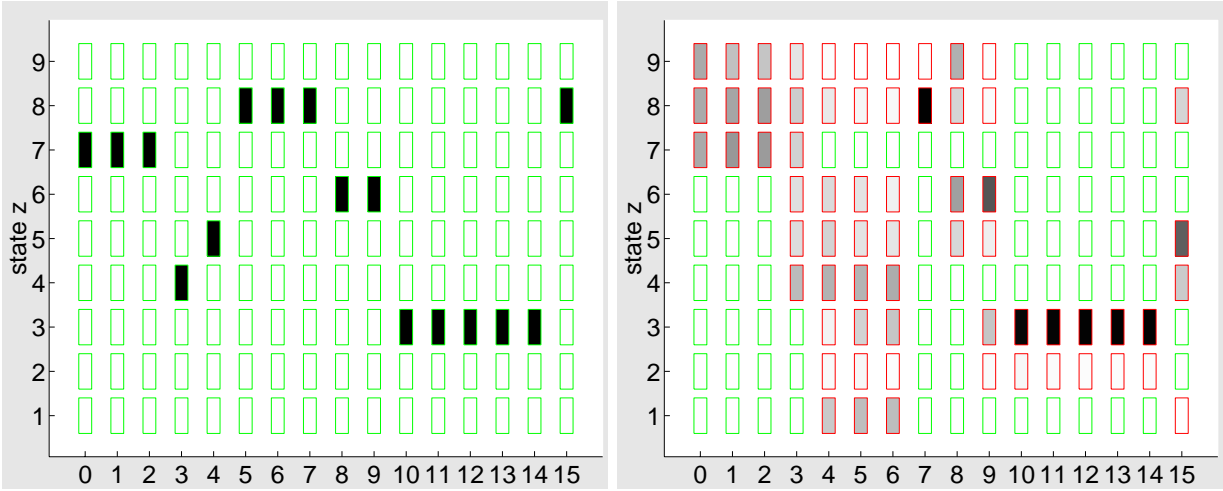
BRIDGE Benchmark problem

Scenario 2

I/O pair:



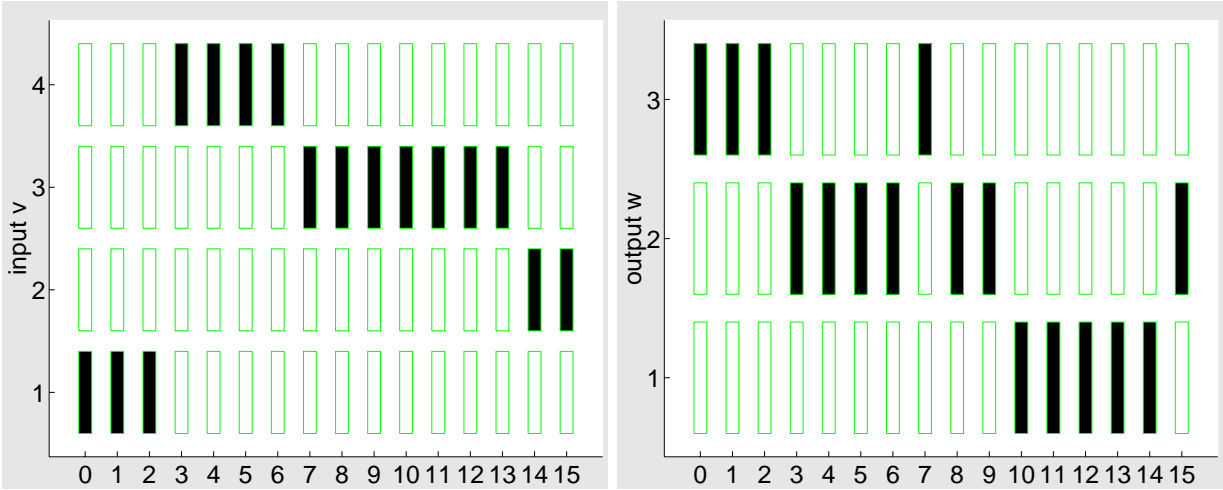
State observation:



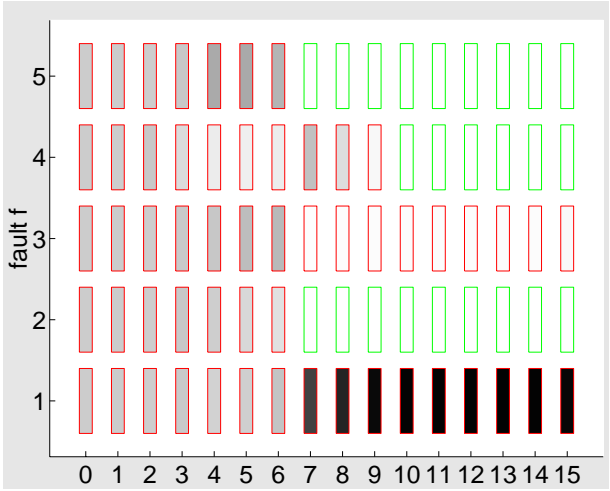
BRIDGE Benchmark problem

Scenario 2

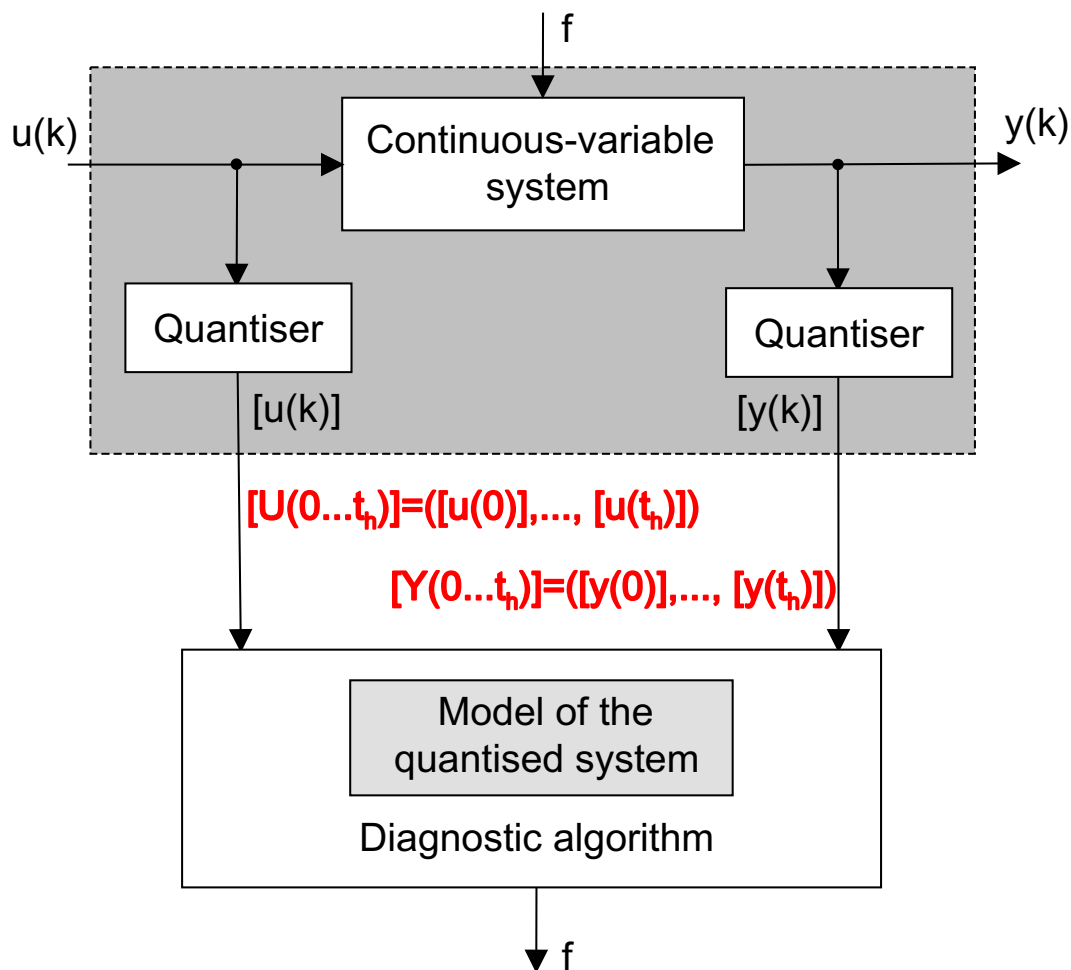
I/O pair:



Solution:



Diagnosis of quantised systems



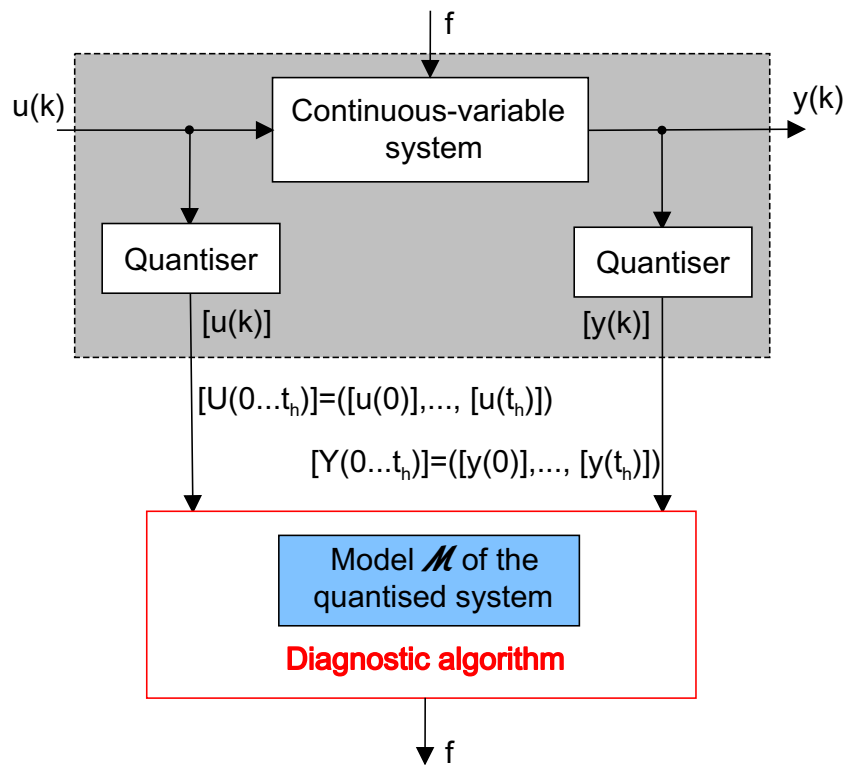
Diagnostic problem:

Given: Quantised system

I/O pair $([U(0...t_h)], [Y(0...t_h)])$

Find: Fault f

Diagnosis of quantised systems



Solution steps

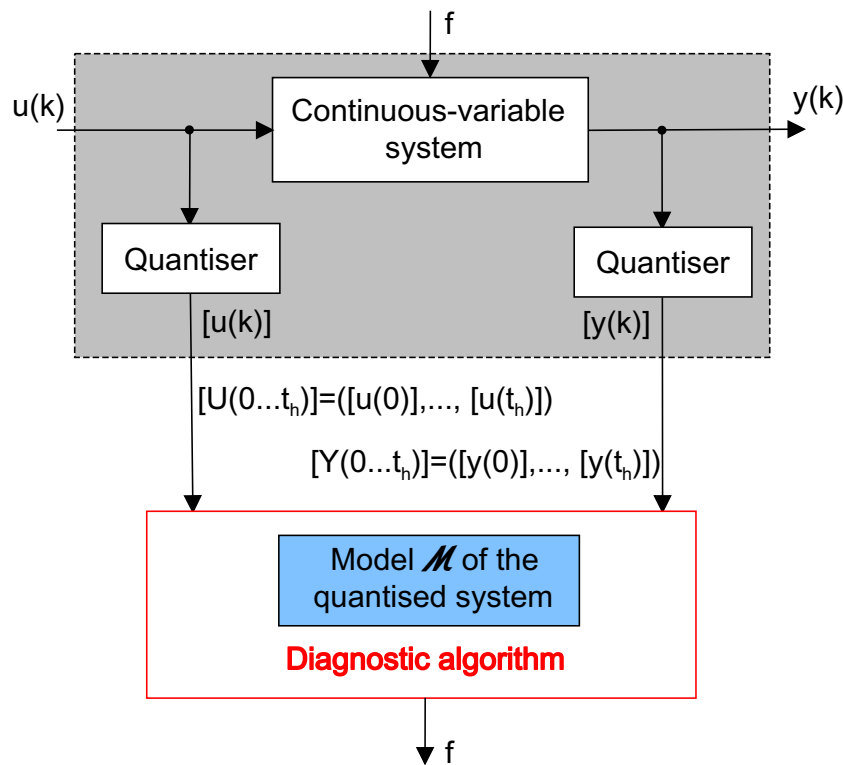
1. Modelling

Determine a complete discrete-event model

2. Diagnosis

Use the diagnostic method for stochastic automata to check whether $([U], [Y])$ is consistent with the discrete-event model

Diagnosis of quantised systems



Diagnostic results:

The results obtained for the discrete-event model hold for the quantised system because the model is complete.

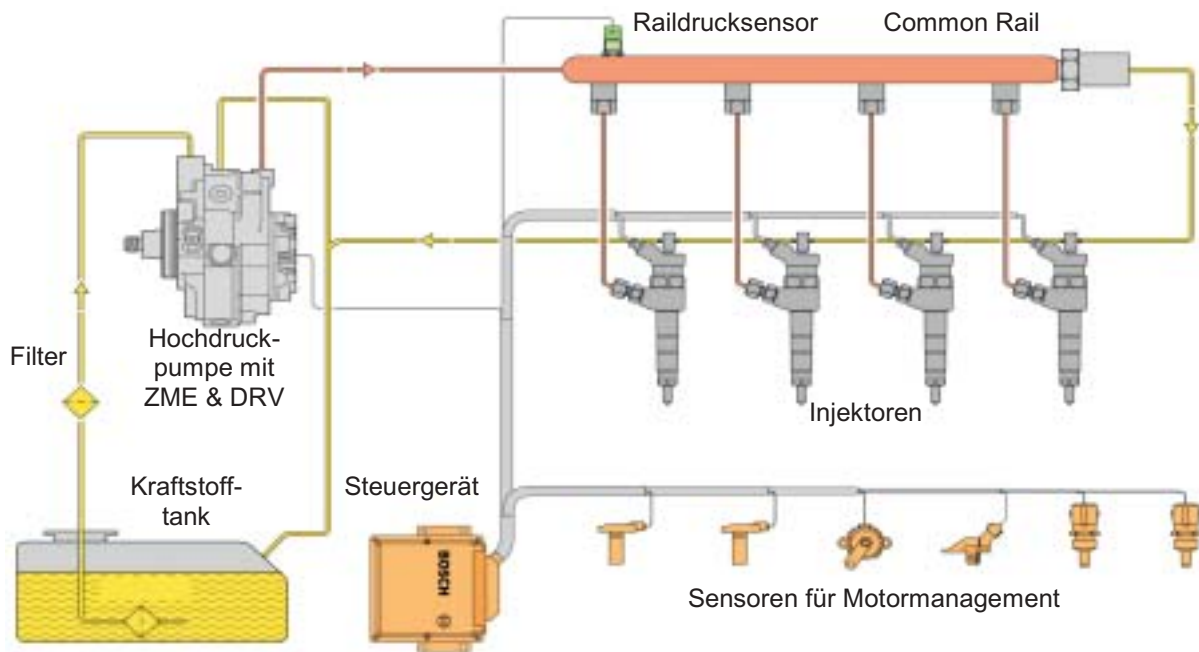
- **Fault detection:**

If $([U], [Y])$ is inconsistent with the model, a fault does exist.

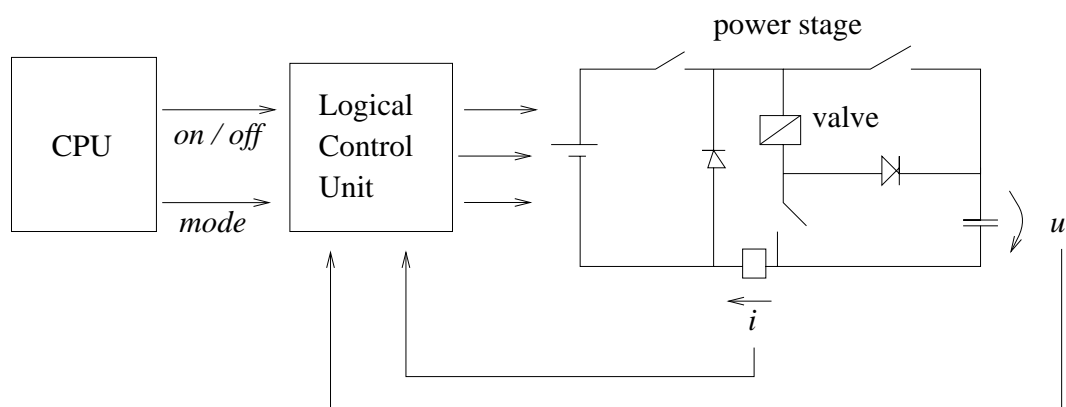
- **Fault identification:**

If $([U], [Y])$ is consistent with the model that holds for fault f , f is a fault candidate.

Example: Diagnosis of the common rail diesel injection system

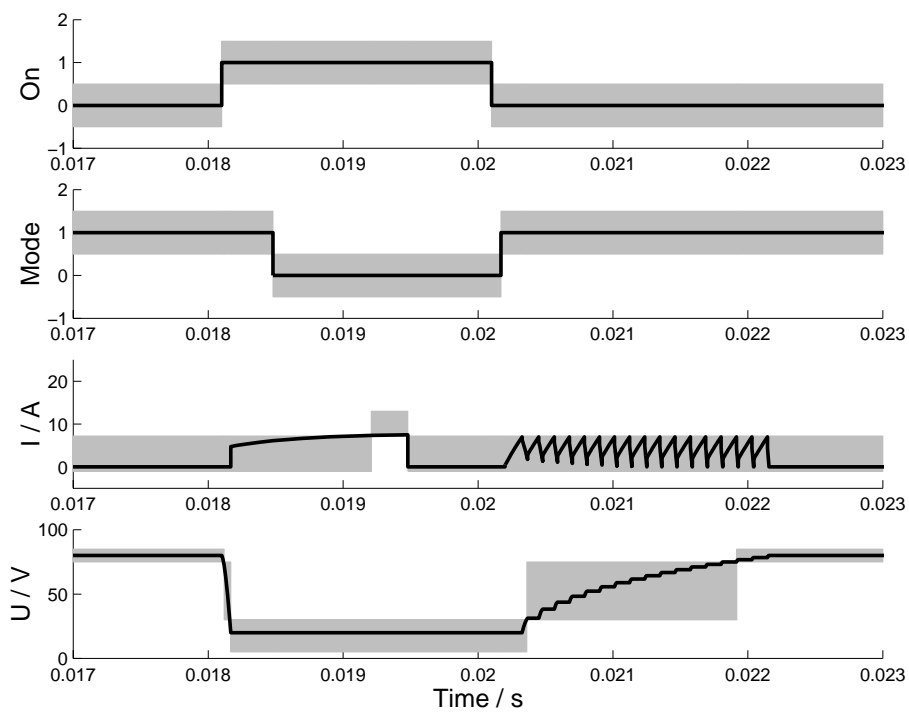


Injector block

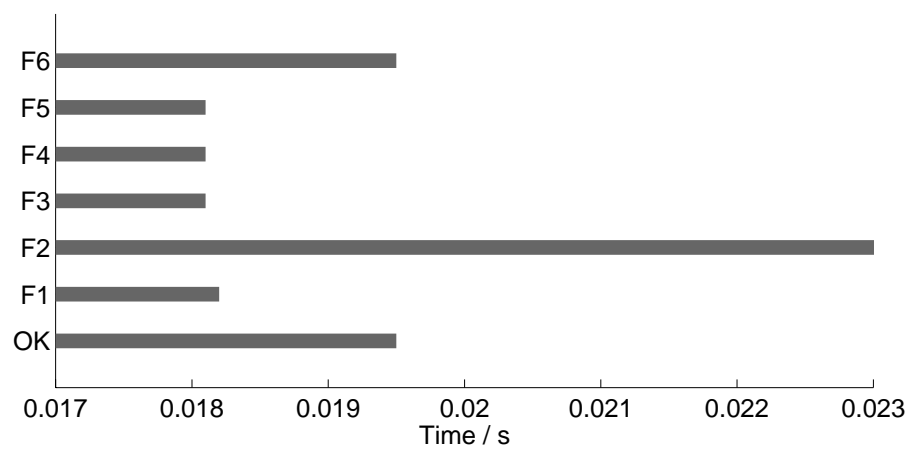


Example: Diagnosis of the common rail diesel injection system

Quantised measurements

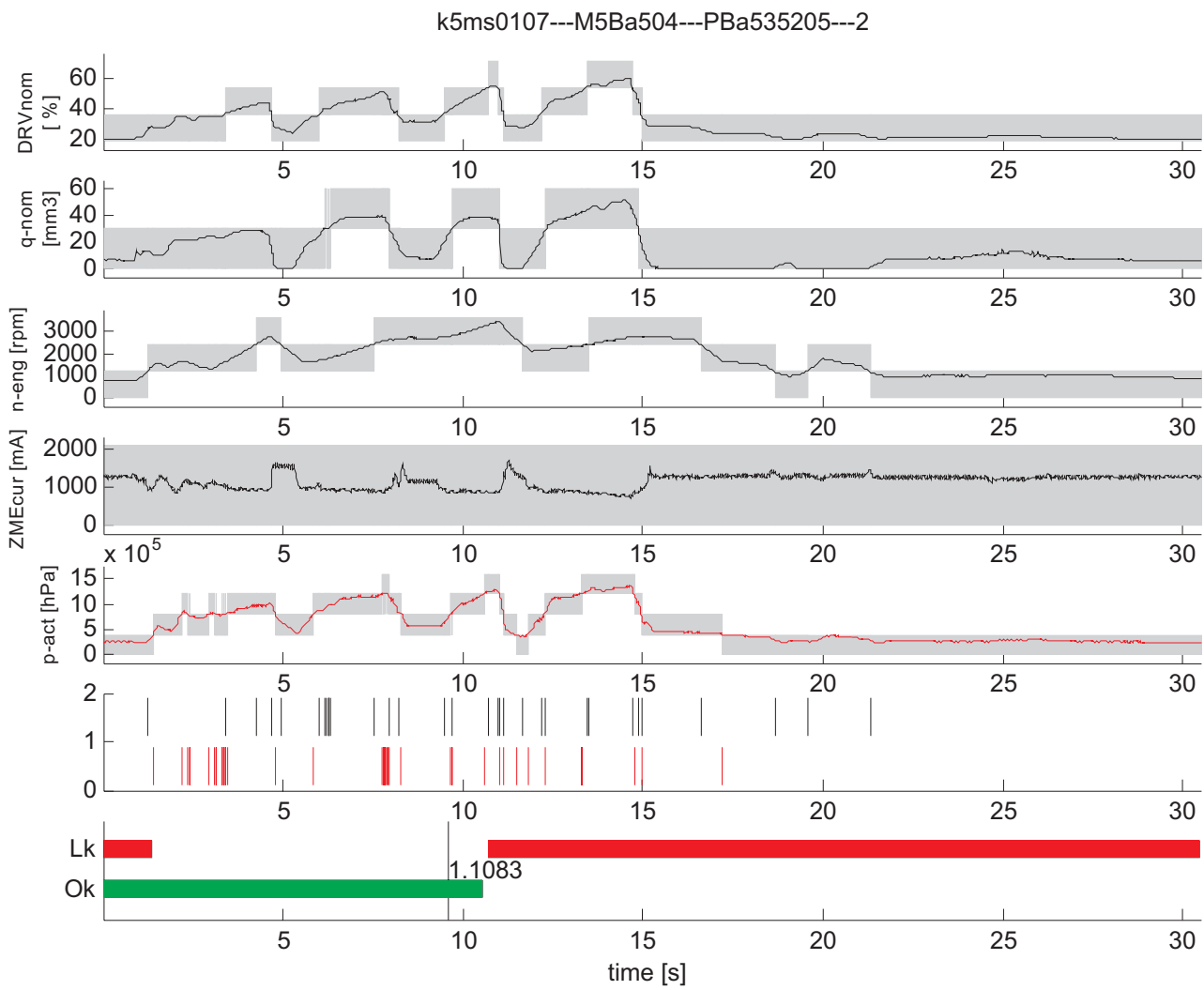


Diagnostic result for the injector

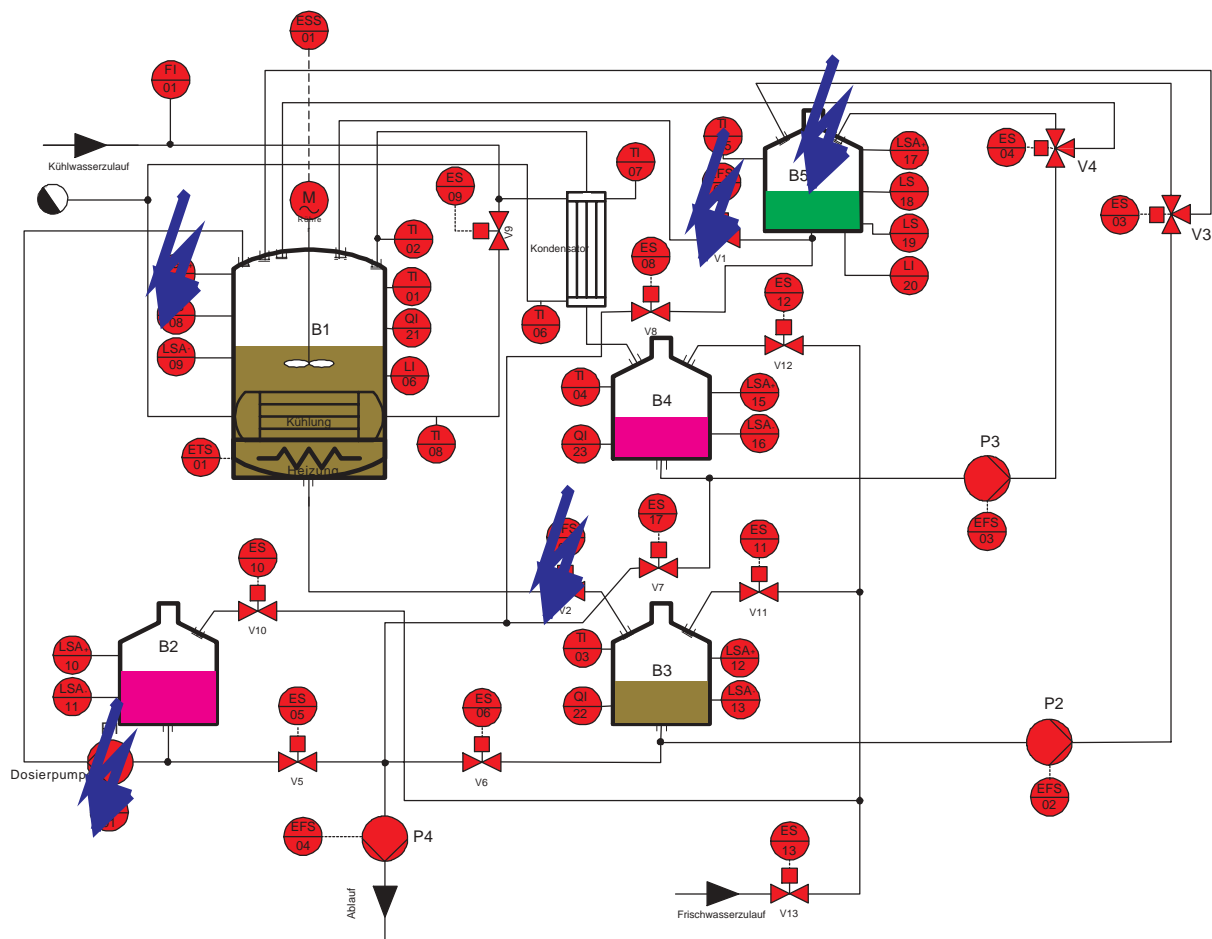


Example: Diagnosis of the common rail diesel injection system

Diagnostic result for the pressure control system:

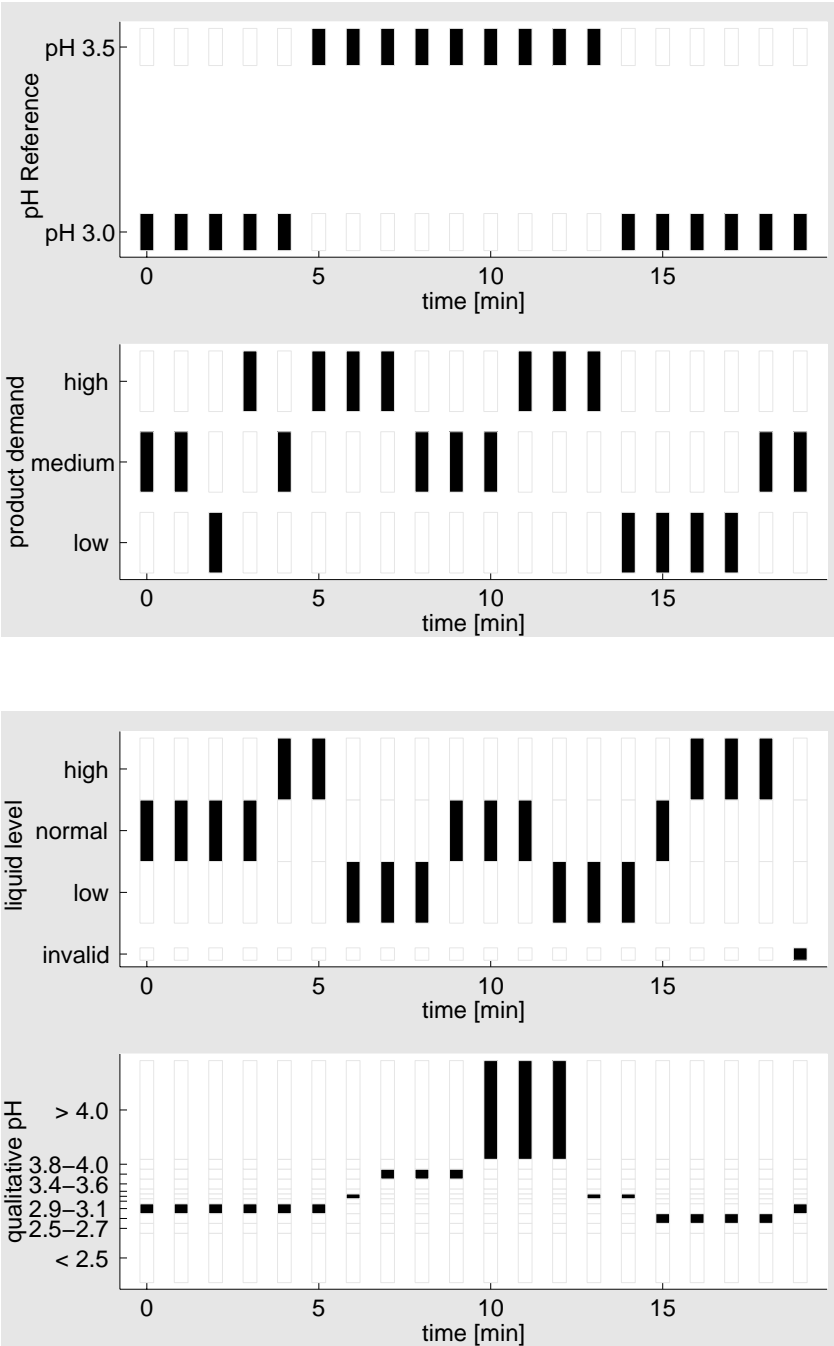


Example: Diagnosis of a neutralisation process



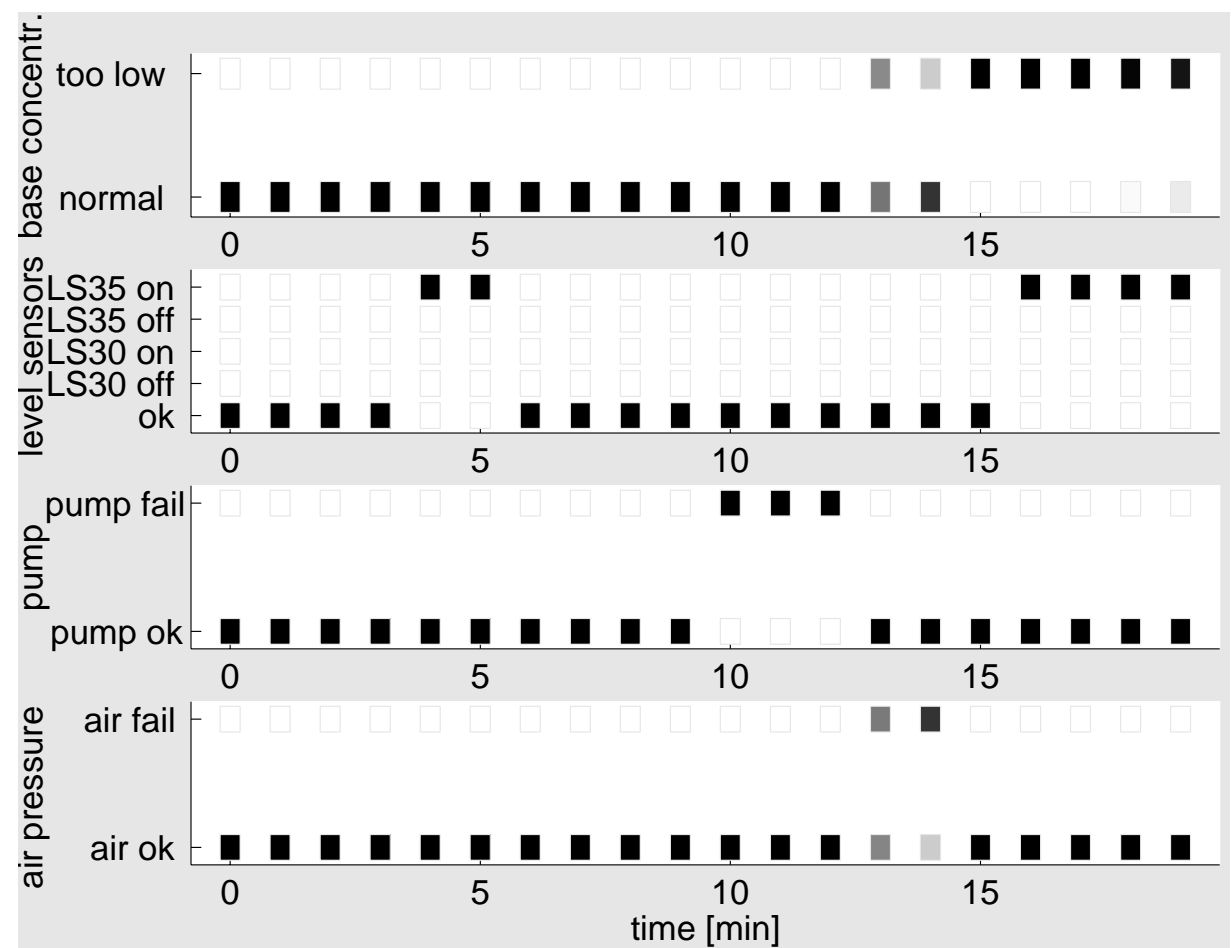
Example: Diagnosis of a neutralisation process

Quantised measurements

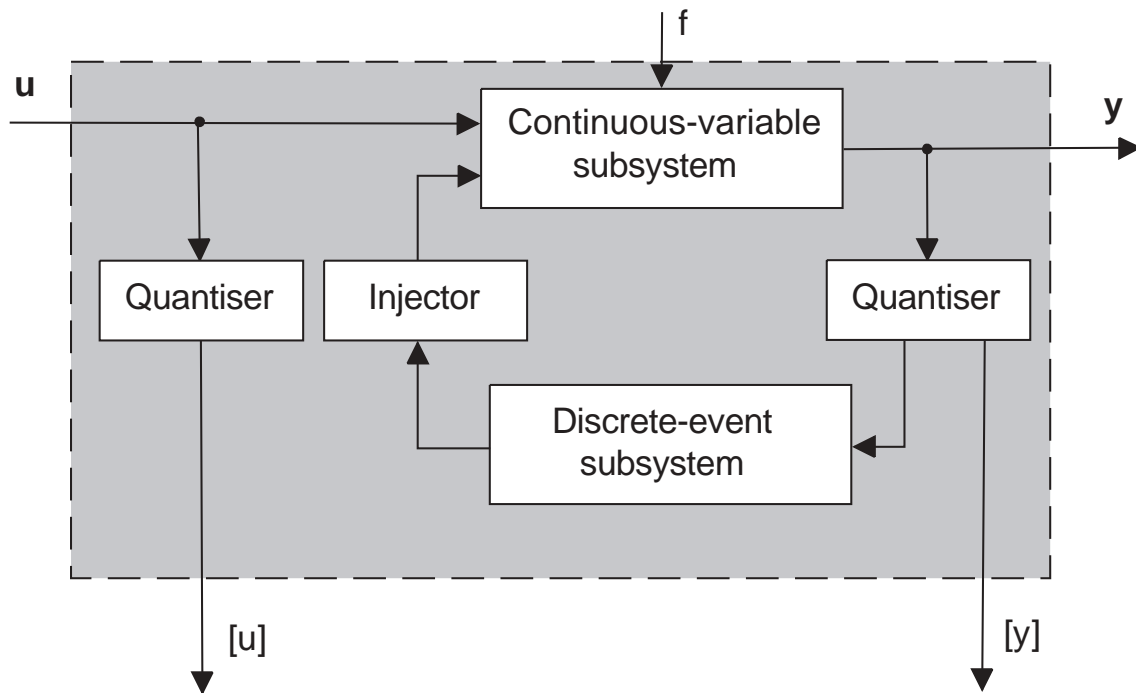


Example: Diagnosis of a neutralisation process

Diagnostic results



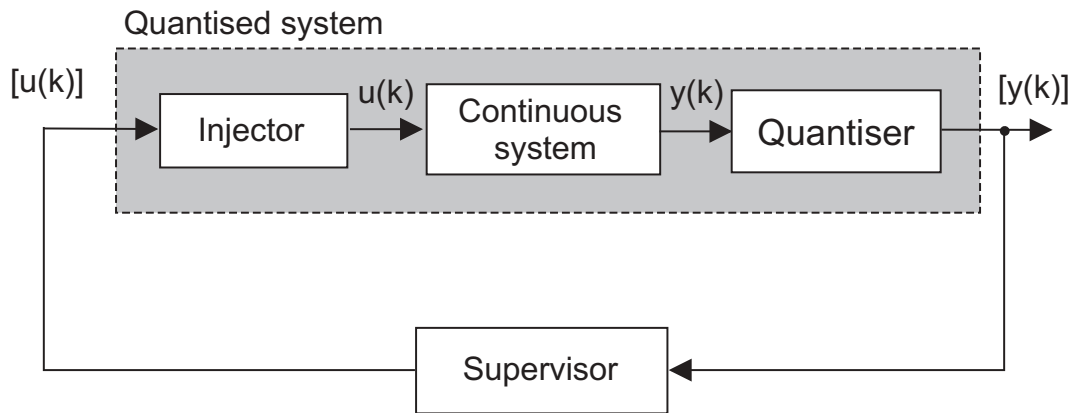
Conclusions



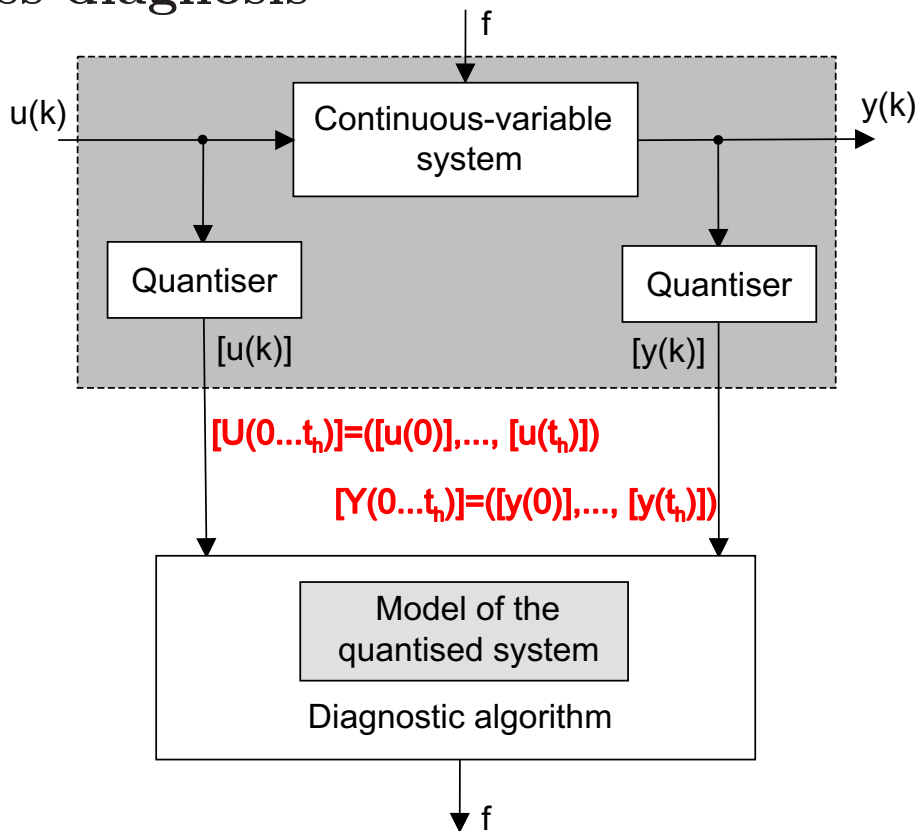
- Quantisers occur naturally within dynamical systems
- Quantisers reflect the uncertainties of the inputs and measurements
- Quantisers are introduced to reduce the information used during the diagnosis
 - Reduce the measurement information
 - Reduce the model complexity

Conclusions

Supervisory control



Process diagnosis



To solve process supervision tasks, ignore as many details as possible: [Use discrete-event models of the hybrid system](#)

Conclusions

- The dynamics of quantised systems can be described by [event sequences](#)
- Complete discrete-event representations of quantised systems can be obtained by [abstraction](#).
- [Diagnosis](#) means to test whether the measured I/O pair is consistent with the model.
- Methods and algorithms are available for diagnosing quantised systems

[The theory on quantised systems bridges the gap between continuous systems theory and discrete systems theory](#)

References

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Discrete models for hybrid systems
in *Modelling, Analysis, and Design of Hybrid Systems*
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